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PRINCIPLES OF ELECTRICAL ENGINEERING

COURSE FILE

Prepared by

MANJUL KHARE

B. RAMESH BABU

M.PRADEEP

POOJA RANI

GEETHANJALI COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF *Electronics and Communication Engineering*

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Prepared by :

1) Name : T.POOJA RANI
2) Sign :
3) Design : Asst. prof.
4) Date : 16/11/2015

1) Name : MANJUL KHARE
2) Sign :
3) Design : Asst. prof
4) Date : 16/11/2015

Verified by :

1) Name :
2) Sign :
3) Design :
4) Date :

* <u>For Q.C Only.</u>

1) Name :
2) Sign :
3) Design :
4) Date :

Approved by : (HOD)

1) Name :
2) Sign :
3) Date :

2.SYLLABUS

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITYHYDERABAD

II Year B.Tech.E CE -II Sem T P C 4+1* 0 4

PRINCIPLES OF ELECTRICAL ENGINEERING

Objectives:

The course introduces the basic concepts of transient analysis of the circuits, the basic two-port network parameters and the design analysis of filters and attenuators and their use in the circuit theory. The emphasis of this course is laid on the basic operation of the DC machines and transformers which includes DC generators and motors, single-phase transformers.

UNIT –I Transient Analysis (First and Second Order Circuits)

Transient Response of RL, RC Series, RLC Circuits for DC excitations, Initial Conditions, Solution using Differential Equations approach and Laplace Transform Method.

UNIT – II – Two Port Networks

Impedance Parameters, Admittance Parameters, Hybrid Parameters, Transmission (ABCD) Parameters, Conversion of one Parameters to another, Conditions for Reciprocity and Symmetry, Interconnection of Two Port networks in Series, Parallel and Cascaded configurations, Image Parameters, Illustrative problems.

UNIT – III – Filters and Symmetrical Attenuators

Classification of Filters, Filter Networks, Classification of Pass band and Stop band, Characteristic Impedance in the Pass and Stop Bands, constant- K Low Pass Filter, High Pass Filter, m-derived T-Section, Band Pass filter and Band Elimination filter, Illustrative Problems.

Symmetrical Attenuators – T Type Attenuator, - Type Attenuator, Bridged T type Attenuator, Lattice Attenuator.

UNIT –IV – DC Machines

Principle of Operation of DC Machines, EMF equation, Types of Generators, Magnetization and Load Characteristics of DC Generators.

DC Motors, Types of DC Motors, Characteristics of DC Motors, Losses and Efficiency, Swinbrune's Test, Speed Control of DC Shunt Motor, Flux and Armature Voltage control methods.

UNIT – V – Transformers and Their Performance

Principle of Operation of Single Phase transformer, Types, Constructional Features, Phasor Diagram on no Load and Load, Equivalent Circuit, Losses and Efficiency of Transformers and Regulation, OC and SC Tests, Predetermination of Efficiency and Regulation (Simple Problems). Synchros, Stepper Motors,.

Text Books :

1. Electric circuits- A.Chakrabarthy, Dhanipat Rai & Sons.
2. Basic concepts of Electrical Engineering- PS Subramanyam, BS Publications

Reference Books :

1. Engineering Circuit Analysis – W.H.Hayt and J. E. Kermmerly and S. M. Durbin 6 ed., 2008 TMH.
2. Basic Electrical Engineering- S.N.Singh, PHI.
3. Electrical Circuits- David A.Bell, Oxford University Press.
4. Electric Circuit Analysis- K.S.Suresh Kumar, Pearson Education.

3.Vision of the Department

To impart quality technical education in Electronics and Communication Engineering emphasizing analysis, design/synthesis and evaluation of hardware/embedded software using various Electronic Design Automation (EDA) tools with accent on creativity, innovation and research thereby producing competent engineers who can meet global challenges with societal commitment.

4.Mission of the Department

i. To impart quality education in fundamentals of basic sciences, mathematics, electronics and communication engineering through innovative teaching-learning processes.

ii. To facilitate Graduates define, design, and solve engineering problems in the field of Electronics and Communication Engineering using various Electronic Design Automation (EDA) tools.

iii. To encourage research culture among faculty and students thereby facilitating them to be creative and innovative through constant interaction with R & D organizations and Industry.

iv. To inculcate teamwork, imbibe leadership qualities, professional ethics and social responsibilities in students and faculty.

5. Program Educational Objectives and Program outcomes of B. Tech (ECE) Program

Program Educational Objectives of B. Tech (ECE) Program :

I. To prepare students with excellent comprehension of basic sciences, mathematics and engineering subjects facilitating them to gain employment or pursue postgraduate studies with an appreciation for lifelong learning.

II. To train students with problem solving capabilities such as analysis and design with adequate practical skills wherein they demonstrate creativity and innovation that would enable them to develop state of the art equipment and technologies of multidisciplinary nature for societal development.

III. To inculcate positive attitude, professional ethics, effective communication and interpersonal skills which would facilitate them to succeed in the chosen profession exhibiting creativity and innovation through research and development both as team member and as well as leader.

Program Outcomes of B.Tech ECE Program:

1. An ability to apply knowledge of Mathematics, Science, and Engineering to solve complex engineering problems of Electronics and Communication Engineering systems.
2. An ability to model, simulate and design Electronics and Communication Engineering systems, conduct experiments, as well as analyze and interpret data and prepare a report with conclusions.
3. An ability to design an Electronics and Communication Engineering system, component, or process to meet desired needs within the realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability and sustainability.
4. An ability to function on multidisciplinary teams involving interpersonal skills.
5. An ability to identify, formulate and solve engineering problems of multidisciplinary nature.
6. An understanding of professional and ethical responsibilities involved in the practice of Electronics and Communication Engineering profession.
7. An ability to communicate effectively with a range of audience on complex engineering problems of multidisciplinary nature both in oral and written form.
8. The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental and societal context.
9. A recognition of the need for, and an ability to engage in life-long learning and acquire the capability for the same.
10. A knowledge of contemporary issues involved in the practice of Electronics and Communication Engineering profession
11. An ability to use the techniques, skills and modern engineering tools necessary for engineering practice.
12. An ability to use modern Electronic Design Automation (EDA) tools, software and electronic equipment to analyze, synthesize and evaluate Electronics and Communication Engineering systems for multidisciplinary tasks.
13. Apply engineering and project management principles to one's own work and also to manage projects of multidisciplinary nature

6. Course objectives and outcomes:

COURSE EDUCATIONAL OBJECTIVES (CEOs)

UNIT	OBJECTIVES
1.	To Know the Transient behavior of First Order and Second Order Circuits
2.	To Know the various parameters of Two Port networks and conditions of parameters and applications
3	To know the different types of Filters and applications
	To know the different types of Attenuators and their applications
4	To know the principle and operation of DC Motor ,constructional design and to find torque and efficiency of a dc motor applications
	To know the constructional details of Generator and able to find the emf and applications
5	To know the Principle and operation of a transformer and phasor diagrams and testing of transformer and to find efficiency of a transformer and applications
	To know the operation and principle of different types of special motors and applications

COURSE OUTCOMES(Cos)

MODULES	OUTCOMES
UNIT-I (Transient Analysis First order and Second order circuits)	
Transient Response of RL circuit	To solve First order circuit of a RL circuit problems w.r.t initial conditions
Transient Response of RCcircuit	To solve First order circuit of a RC circuit problems w.r.t initial conditions
RLC Circuit	To solve second order circuit by differential equation approach method for given initial conditions
Laplace transform method	Students can able to solve first order and second order circuits using Laplace transform method
UNIT –II(Two Port Network)	
Impedance and Admittance parameters	Students can able to find the impedance and admittance of given circuit and their condition
Pas band and Stop band filter	Ability to design pas band and stop band filters and their applications
Conversion of one parameter to another parameter	Students can able to convert one parameter to another parameter and also solve the twoport network problems
Condition for Reciprocity and Symmetry	Ability to get condition for reciprocity and symmetry for different parameters

Interconnection of networks	Students can able to design of different networks and able to find parameters for seires,parallel,cascaded networks
UNIT –III(Filters & Symmetrical Attenuators)	
Classification of filters and networks	Students can identify different types of filters and their classification
Alternating Quantities	Students can identify and analyze the different types of alternating quantities and importance
Phasor diagrams	Ability to draw phasor diagrams for different types of ac networks and relationship between the quantities
Series circuits	Design the series circuit and solve the circuit problems
Symmetrical Attenuators	Students can identify and analyze the different type of Attenuators
T-type Attenuator	Ability to Design the T-Attenuator and its use
Pie -Attenuator	Ability to Design the Pie-Attenuator and its use
Bridge type Attenuator	Ability to Design the Bridge-Attenuator and its use
UNIT-IV(DC MACHINES)	
Operation and Construction	Design and Construction of a dc generator and principle of operation
Types of generators	Students can able to know different types of generators and their functions
EMF equation of generator	Ability to derive EMF equation and calculate EMF for given parameters
Principle of operation of dc motor	Students can understand the principle and operation of dc motor
Construction	Design and construction of a dc Motor
Types of DC motor	Students can able to know different types of motors and their functions
Torque	Function of torque and importance and ability to calculate torque for given parameters
Losses	Students can know the different types of losses in dc motor
Efficiency and problems	Ability to find efficiency of a different types of dc motor

UNIT-V (Transformers & THEIR PERFORMANCE)	
Principle of operation and construction	Design and Construction of the transformer and operation
Losses	Students can able to know the different types of losses and their role
Practical and ideal transformer	Ability to find difference between ideal and practical transformer and their importance
Transformer Tests	Ability to determine the losses i.e core losses and copper losses
Efficiency	Ability to find the efficiency of transformer for different loads in real time applications
Regulation and problems	He can know what is regulation and ability to solve regulation problems and importance
Synchros	Helps the students to analyse the basic concepts of DC machine in the working of some special AC machines

Stepper Motors	Helps the students to analyse the basic concepts of DC machine in the working of some special AC machines and their applications

7. Brief notes on the importance of the course and how it fit into the curriculum

The course introduces the basic concepts of transient analysis of the circuits, the basic two-port network parameters and the design analysis of filters and attenuators and their use in the circuit theory. The emphasis of this course is laid on the basic operation of the DC machines and transformers which includes DC generators and motors, single-phase transformers.

8. Prerequisites

Engineering Physics, Mathematics

9. Instructional Learning Outcomes:

Outcomes

On successful completion of this subject, students will be able to:

1. Understand working principles of electrical devices and circuits.
2. Understand advantages & applications of electrical devices and circuits.
3. Understand design and analysis of electrical circuits.
4. To apply the operating knowledge of major electrical devices like DC generator, DC motor, Transformers, Syncro transmitter & receiver and advanced filter and attenuator circuits to identify, formulate & solve Engineering problems by making use of modern software/hardware tools.

10.Course mapping with PEOs and Pos:

a) an ability to apply the knowledge of Mathematics, science and engineering in Electronics and communications	
b) an ability to Design & Conduct Experiments, as well as analyze & Interpret Data	√
c) an ability to design a system, component, or process to meet desired needs with in realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability	√
d) an ability to function on multidisciplinary teams	√
e) an ability to Identify, Formulate & Solve problems in the area of Electronics and Communications Engineering	√
f) an understanding of professional and ethical responsibility	
g) an ability to communicate effectively	

h) the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context	
i) a recognition of the need for, and an ability to engage in life-long learning	√
j) a knowledge of contemporary issues	
k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice	√

Relationship of the course to the program educational objectives :

1. **Domain knowledge:** Graduates will be able to synthesize mathematics, science, engineering fundamentals, laboratory and work-based experiences to formulate and solve engineering problems in Electronics and Communication engineering domains and shall have proficiency in Computer-based engineering and the use of computational tools.	√
2. **Professional Employment:** Graduates will succeed in entry-level engineering positions within the core Electronics and Communication Engineering, computational or manufacturing firms in regional, national, or international industries and with government agencies.	√
3. **Higher Degrees:** Graduates will succeed in the pursuit of advanced degrees in Engineering or other fields where a solid foundation in mathematics, science, and engineering fundamentals is required.	√
4. **Engineering citizenship:** Graduates will be prepared to communicate and work effectively on team based engineering projects and will practice the ethics of their profession consistent with a sense of social responsibility.	√
5. **Lifelong Learning:** Graduates will recognize the importance of, and have the skills for, continued independent learning to become experts in their chosen fields and to broaden their professional knowledge.	√
6. **Research and Development:** To undertake Research and Development works in the areas of Electronics and Communication fields.	√

11.Class time table

To be attached

12: Individual TIME TABLE

To be attached

13. Lecture schedule with methodology being used/adopted

Unit wise Summary

S.No.	Unit No.	Total no. of Periods	Topics to be covered	Reg / Additional	Teaching aids usedLCD/ OHP/BB	Remarks
1	I	15	Introduction, DC Excitation	Regular	BB	
			RL Series ckt (DC)			
			RC series ckt (DC)			
			RLC series ckt (DC)			
			Numerical problems			
			Laplace Transformation			
			Problems			
			Additional topic			
			tutorial			
			Unit revision, Objective questions, Assignment			
2	II	15	. Two port Networks	Regular	BB	
			. Impedance parameters, problems			
			. Admittance Parameters, problems			
			. Hybrid Parameters, problems			
			. ABCD Parameters, problems			
			. Conversion of parameters, problems			
			. Condition for symmetry and reciprocity			
			. Interconnection of ports (Series & Parallel)			
			. Cascade configuration, Image parameters			
			. Additional topic			
			. tutorial			
			. Numerical problems, Objective questions, Assignment			
3	III	15	. Filter Introduction, Classification of Filter's	Regular	BB	
			. Filter Networks, Pass and stop band			
			. constant k low pass filter, High pass filter			
			. M –derived T and Pi section filters			
			. Band pass filter			
			. Band Elimination filter			
			. Numerical problems,			
			. Symmetrical Attenuators			
			. T-Type Attenuators			

			.pi-Type Attenuators			
			.Bridged T Attenuators			
			.Lattice Attenuators			
			.Additional topic			
			.tutorial			
			.Basic Problems, Objective questions, Assignment			
4	**IV**	**15**	.Introduction to DC machines	Regular	BB	
			.Principle of operation of a simple loop generator			
			.Action of Commutator & EMF Equation			
			.Types of DC Generators			
			.Magnetization characteristics of DC Generators			
			.Load characteristics of DC Generators			
			.Applications, Numerical Problems			
			.DC Motors Introduction, back emf and its significance			
			.Types of DC Motors			
			.Characteristics of DC Motors			
			.Losses and efficiency			
			.Torque equations			
			.Speed control of DC motor, Testing of DC machines			
			.Numerical problems			
			.Additional topic			
			.tutorial			
			.Unit revision, Objective questions, Assignment			
5	**V**	**15**	.Transformers Introduction	Regular	BB	
			.Principle of Operation			
			.Construction & types			
			.Phasor diagrams, Equivalent circuits			
			.Transformer losses, efficiency			
			.Regulation, OC & SC tests			
			.Numerical Problems			
			. Synchro's & Stepper Motors			
			. Additional topic			
			.tutorial			
			.Unit revision, Objective questions, Assignment			

14. DETAILED NOTES

Unit - I

①

Transient Analysis (Response of RL & RC & RLC Circuits in Series)

A Network in which branch currents and Node voltages are not changing with respect to time is said to be steady state. In other words, if the voltages and currents in the circuit are having a constant Amplitude and frequency throughout the time interval of these parameters (current & voltages) measurement, then such a Network is said to be in steady state.

Whenever a network is switched from one condition to another by change in applied voltage (or) by change in one of the circuit elements, in a period of time, branch currents and voltages change from their former values to New one. This time interval is called transition period. The response (or) output of network during transition period is called transient response of Network.

After this transition period, if the network condition is not disturbed, then the network attains steady state at infinite time.

The application of laws to the Network containing energy storing elements such as inductor and capacitor, results in a differential equation whose solution consists of two parts, the complementary function and Particular function. The complementary function represents transient part of solutions which decays with time, while remaining term represents steady state part of solution.

Initial conditions of the elements in the network must be known to evaluate arbitrary constants in the general solution of differential equation. In the analysis of networks, behaviours of elements individually and in combinations is studied with initial conditions.

Mathematical Background of Differential Equations :-

n^{th} order differential equation is expressed as,

$$a_0 \frac{d^n i}{dt^n} + a_1 \frac{d^{n-1} i}{dt^{n-1}} + \text{-----} + a_{n-1} \frac{di}{dt} + a_n \cdot i = V(t)$$

where $a_0, a_1, \text{----}, a_{n-1}$ & a_n are constants

Here $i(t)$ is a dependent variable and w.r.t an electrical circuit, generally current dependent on the voltage applied.

$V(t)$ is the independent variable and w.r.t electrical circuit. Voltage is called Input (or) Forcing Function (or) Excitation.

The Solution of the equation is called as the response of the System.

For the First order differential Equation having $n=1$ above eq becomes,

$$a_0 \frac{di(t)}{dt} + a_1 i(t) = V(t)$$

This equation may further be classified as homogeneous & Non-homogeneous differential equations.

Homogeneous equation is the one in which Forcing Function, $V(t)$ is zero and is of the form,

$$a_0 \frac{d\,i(t)}{dt} + a_1 i(t) = 0 \rightarrow \text{Homogeneous Equation.}$$

While a Non-Homogeneous Equation is a linear differential equation which involves the function $V(t)$ which is independent variable of the system.

It is of the form,

$$a_0 \frac{di(t)}{dt} + a_1 i(t) = V(t) \rightarrow \text{Non-Homogeneous Equation}$$

General and Particular Solutions:-

Homogeneous Equation :-

Consider a first order homogeneous linear differential eq:

$$a_0 \frac{di(t)}{dt} + a_1 i(t) = 0$$

To find solution of $i(t)$,

$$\Rightarrow \quad a_0 \frac{di(t)}{dt} = -a_1 i(t)$$

$$\frac{di(t)}{dt} = \frac{-a_1}{a_0} i(t)$$

By Integrating the equation

$$\int \frac{di(t)}{i(t)} = \int \frac{-a_1}{a_0} d(t) + K_1 \quad \text{where } K_1 \text{ is Integration constant.}$$

$$\therefore \ln(i) = \frac{-a_1}{a_0} \cdot t + K_1$$

$$\Rightarrow \ln(i) = \ln\left[e^{-\frac{a_1}{a_0}t}\right] + \ln K \quad \text{where } K_1 = \ln K$$

As we know $\ln x + \ln y = \ln xy$

$$\therefore \ln(i) = \ln\left[K \cdot e^{-\frac{a_1}{a_0}t}\right]$$

$$\Rightarrow \boxed{i = K \cdot e^{-\frac{a_1}{a_0} \cdot t}}$$

If K_1 is unknown & is an arbitrary constant, then the solution is called General Solution. But if some additional information about the network is known, then K_1 can be evaluated, for which the solution is called particular solution.

Eg:- If $t = 0$ & $i(t) = a_2$, then $i = K e^{-\frac{a_1}{a_0} \times 0} = a_2$

$$\Rightarrow K = a_2$$

$\therefore$ Particular solution is $\Rightarrow i = a_2 \cdot e^{-\frac{a_1}{a_0} \cdot t}$

Non-Homogeneous Equation :-

Consider a Non-homogeneous equation of first order,

$$\frac{di}{dt} + Pi = Q$$

This eq may be obtained by rearranging the variables and defining the new constant P. The variable Q may (or) may not be the function of time.

To find the solution and find value of i, multiply both sides of the equation by integrating factor e^{Pt}.

We get,

$$e^{Pt} \cdot \frac{di}{dt} + Pi\,e^{Pt} = Q \cdot e^{Pt}$$

As we know

$$d(xy) = x\,dy + y\,dx$$

Let, $x = i$ & $y = e^{Pt}$

$$\therefore \frac{d}{dt}\left(i \cdot e^{Pt}\right) = e^{Pt} \cdot \frac{di}{dt} + i \cdot e^{Pt} \cdot P$$

Thus we have $\frac{d}{dt}\left(i \cdot e^{Pt}\right) = Q \cdot e^{Pt}$

Integrating both sides,

$$i\,e^{Pt} = \int Q\,e^{Pt}\,dt + K$$

$$\therefore \boxed{i = e^{-Pt}\int Q \cdot e^{Pt}\,dt + K \cdot e^{-Pt}}$$

The first term in the above equation is called as Particular Integral while the second term is known as complementary function.

A General solution can be written as,

$$i = i_{PI} + i_{CF}$$

where i_{PI} – Particular Integral
i_{CF} – Complementary Integral

i_{PI} may be written as steady state value &
i_{CF} is called as transient position.

$$\boxed{\therefore i = i_{ss} + i_t}$$

i_{ss} is the one which remains as $t \to \infty$ & i_t eventually vanishes.

Initial conditions in Network:-

We assume that at reference time $t = 0$, network condition is changed by switching action. Assume that switches operate in zero time. The network conditions at this instant are called initial conditions in network. To distinguish between the time just before and just immediately after the condition of network is changed, we will use (−), negative and (+), Positive signs. $t(0^-)$ & $t(0^+)$.

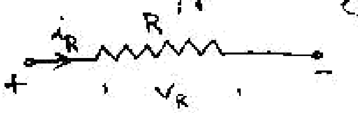
Thus, $t(0^-)$ is the instant at which the condition of network is not yet changed, but it is about to be changed, while $t(0^+)$ is the instant at which the condition of network is just changed. Similarly $i(0^-)$, $V(0^-)$ & $i(0^+)$, $V(0^+)$ etc.,

Initial conditions in the network depends on past or history condition before instant $t = 0^-$. These conditions at $t = 0^-$ are given by voltage across capacitor and current through inductor.

Initial conditions in Elements:-

1) Resistor:-

For resistor having value R, the relation between applied voltage and resulting current is given by

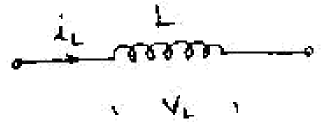


$$\boxed{V = i \cdot R}$$

This equation is linear and time independent. ie, current in the resistor changes instantaneously if applied voltage changes instantaneously. This is because, there is no storage of energy in resistor.

2) Inductor:-

The Relation between current flowing through the inductor and voltage across it is given by,



$$V_L = L\frac{di_L}{dt}$$

If DC current is passed through inductor, $\frac{di_L}{dt}$ becomes zero, hence voltage across inductor, V_L also becomes zero. Thus, as far as dc quantities are considered, steady state, inductor acts as a short circuit.

current in an inductor w.r.t voltage can be expressed as

$$i_L = \frac{1}{L}\int V_L\, dt$$ Here Limits are decided by past history ie., $-\infty$ to $t(0^-)$.

If we consider that switching takes place at $t=0$, we can split limits into two intervals as $-\infty$ to 0 & 0 to t.

Note 0^- is the instant just before switching action takes place,
0^+ is the instant just after switching action takes place.

Hence $$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L \, dt$$

$$\Rightarrow \quad i_L = \underbrace{\frac{1}{L}\int_{-\infty}^{0^-} V_L \, dt}_{\uparrow} + \frac{1}{L}\int_{0^-}^{t} V_L \, dt$$

Initial condition of i_L ($i_L(0^-)$)

$$\Rightarrow \quad i_L = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} V_L \, dt$$

At $t=0^+$
$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} V_L \, dt$$

As we assumed that transient period is zero, integration of 0^- to 0^+ is zero.

$$\therefore \quad \boxed{i_L(0^+) = i_L(0^-)}$$

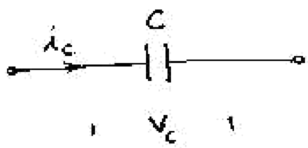
Thus in an inductor "current cannot change instantaneously."
Current in the inductor before and after switching action is same.

At the time of switching, voltage across inductor is ideally ∞ as dt (time interval) is zero. Thus at the time of switching, inductor acts as open circuit. while in steady state at $t=\infty$, it acts as short circuit.

If inductor carries an initial current of I_0 before switching action, then at instant $t=0^+$, it acts as a constant current source of value I_0, while in steady state at $t=\infty$, it acts as a short circuit across a current source.

3) Capacitor :-

Relation between current through capacitor & Voltage across it is given by



$$i_c = C\frac{dV_c}{dt}$$

if DC voltage is applied to capacitor, $\frac{dV_c}{dt}$ becomes zero, (V constant w.r.t time)

Hence current through capacitor i_c becomes zero. Thus capacitor acts as open circuit as far as dc quantities are considered.

Voltage across a capacitor may be expressed as

$$V_c = \frac{1}{C}\int i_c\,dt$$

Limits may be written from past history ie., $-\infty$ to t.

$$\therefore \quad V_c = \frac{1}{C}\int_{-\infty}^{t} i_c\,dt$$

$$\Rightarrow \quad V_c = \underbrace{\frac{1}{C}\int_{-\infty}^{0^-} i_c\,dt}_{} + \frac{1}{C}\int_{0^-}^{t} i_c\,dt$$

↑ Initial Voltage on Capacitor $V_c(0^-)$

$$\therefore \quad V_c = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i_c\,dt$$

At $t = 0^+$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c\,dt$$

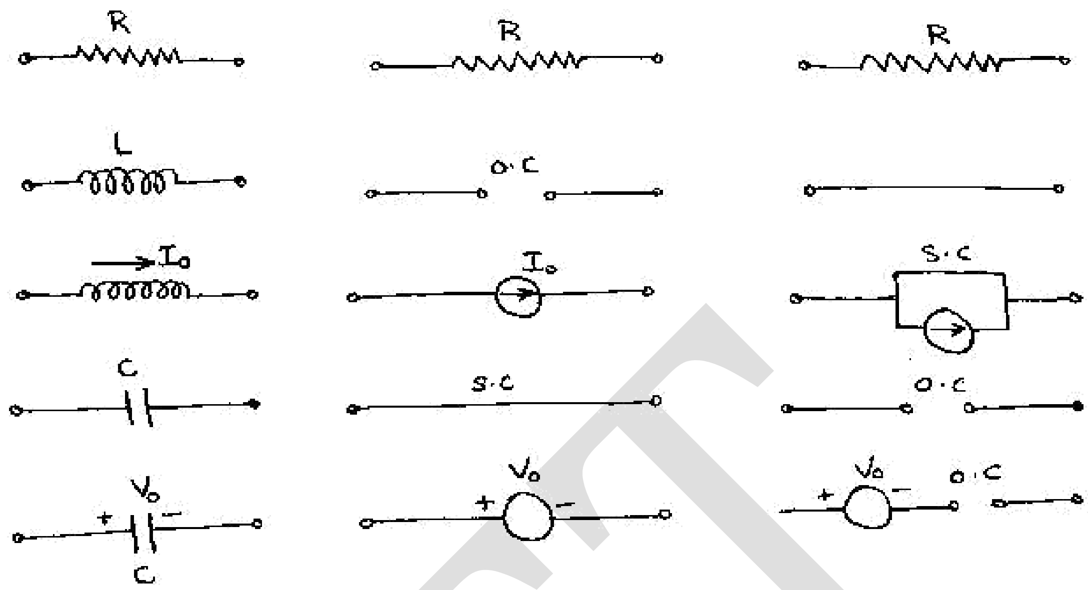
As the transient period is zero, integration from 0^- to 0^+ is zero.

$$\therefore \quad \boxed{V_c(0^+) = V_c(0^-)}$$

Thus "Voltage across capacitor cannot change instantaneously."

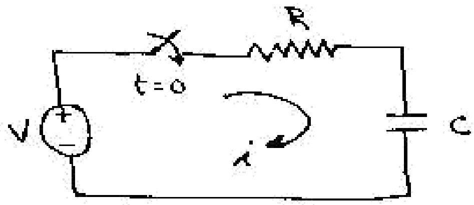
At $t = 0^+$ (for a uncharged capacitor) capacitor acts as short circuit. once it get charged at $t = \infty$, in steady state, it acts as open circuit. Initially capacitor is charged to voltage 'V_0' before switching, then at instant $t = 0^+$ It acts as constant Voltage source of value "V_0", while in steadystate at $t = \infty$, Tabulated Summary :-

Element	Behaviour immediatly after excitation is given $t = 0^+$ instant	Behaviour as $t \to \infty$ ie steady State



Problems

(1) In the network shown, switch K is closed at $t=0$ with the capacitor uncharged. Find the values of $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$, for element values as follows; $V = 100V$, $R = 1000\Omega$, $C = 1\mu F$



Solution:- At $t=0^-$ switch K is open

Hence ct in the circuit is zero. Also as the capacitor is uncharged, so voltage across capacitor is zero.

$$V_c(0^-) = 0 = V_c(0^+) \longrightarrow (1)$$

For $t \geq 0^+$, switch K is closed.

Applying KVL,

$$i \cdot R + \frac{1}{C}\int_{-\infty}^{t} i\,dt = V$$

$$i \cdot R + \underbrace{\frac{1}{C}\int_{-\infty}^{0} i\,dt}_{\text{Initial voltage on capacitor} = 0} + \frac{1}{C}\int_{0}^{t} i\,dt = V$$

$$\Rightarrow \quad i \cdot R + \frac{1}{C}\int_{0}^{t} i\,dt = V \longrightarrow (2)$$

At $t=0^+$, equation becomes

$$i(0^+) \cdot R + \underbrace{\frac{1}{C}\int_{0}^{0} i\,dt}_{=0} = V$$

$$i(0^+) \cdot 1000 + 0 = 100$$

$$i(0^+) = 0.1\,A$$

Now differenciating eq (2) w.r.t t,

$$R \cdot \frac{di}{dt} + \frac{1}{C} \cdot i = 0 \longrightarrow (3)$$

At $t=0^+$ equation becomes

$$R \cdot \frac{di}{dt}(0^+) + \frac{1}{C} \cdot i(0^+) = 0$$

by substituting values, $1000 \cdot \frac{di}{dt}(0^+) + \frac{1}{1 \times 10^{-6}} \cdot (0.1) = 0 \Rightarrow \frac{di}{dt}(0^+) = -100\,A/sec$

Now differenciating equation ③ w.r.t t

$$R \cdot \frac{d^2 i}{dt^2} + \frac{1}{C} \cdot \frac{di}{dt} = 0 \longrightarrow ④$$

At $t = 0^+$; equation ④ becomes

$$R \cdot \frac{d^2 i}{dt^2}(0^+) + \frac{1}{C} \cdot \frac{di}{dt}(0^+) = 0$$

by Substituting Values,

$$1000 \cdot \frac{d^2 i}{dt^2}(0^+) + \frac{1}{1 \times 10^{-6}} \cdot (-100) = 0$$

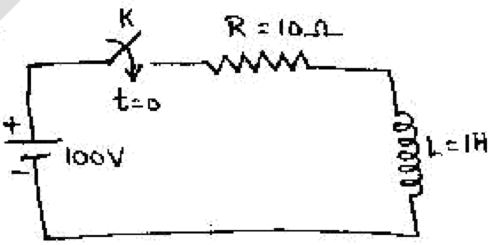
$$1000 \cdot \frac{d^2 i}{dt^2}(0^+) = \frac{100}{1 \times 10^{-6}}$$

$$\Rightarrow \frac{d^2 i}{dt^2}(0^+) = 10^5 \ A/sec^2$$

∴ At $t = 0^+$

$$i(0^+) = 0.1A \ ; \ \frac{di}{dt}(0^+) = -100 A/sec \ ; \ \frac{d^2 i}{dt^2}(0^+) = 10^5 A/sec^2$$

② The switch is closed at $t = 0$; find the value of i, $\frac{di}{dt}$, $\frac{d^2 i}{dt^2}$ at $t = 0^+$. Assume initial current of inductor is zero.



Solution:- At $t = 0^-$, Switch is open.

Hence current in circuit is zero.

$$i(0^-) = 0 = i(0^+) \longrightarrow ①$$

← because current through inductor cannot change instantaneously.

for $t \geq 0^+$, Switch is closed.

Applying KVL,

$$i \cdot R + L \frac{di}{dt} = V \longrightarrow ②$$

At $t = 0^+$, equation becomes,

$$i(0^+) \cdot R + L \frac{di}{dt}(0^+) = V$$

by Substituting the values,

$$0 \cdot R + 1 \cdot \frac{di}{dt}(0^+) = 100$$

$$\frac{di}{dt}(0^+) = 100 \ A/sec$$

Differenciating equation ② wrt t

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} = 0 \longrightarrow ③$$

At $t=0^+$, equation ③ becomes,

$$R\cdot\frac{di}{dt}(0^+) + L\cdot\frac{d^2i}{dt^2}(0^+) = 0$$

$$L\cdot\frac{d^2i}{dt^2}(0^+) = -R\frac{di}{dt}(0^+)$$

$$\frac{d^2i}{dt^2}(0^+) = \frac{-R}{L}\frac{di}{dt}(0^+)$$

by substituting the values,

$$\frac{d^2i}{dt^2}(0^+) = \frac{-10}{1}\times 100 = -1000\ A/sec^2$$

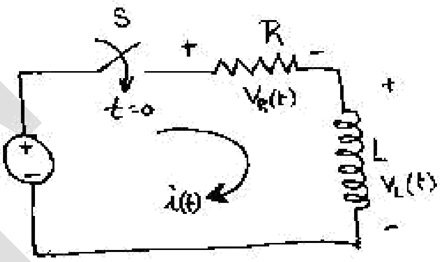
$\therefore$ At $t=0^+$,

$$i(0^+) = 0A;\ \frac{di}{dt}(0^+) = 100\ A/sec;\ \frac{d^2i}{dt^2}(0^+) = -1000\ A/sec^2$$

Transient Response of Series R-L Circuit for DC Excitation:

The inductor in the series R-L circuit may be initially charged (or) uncharged. A series R-L circuit in which an active source is introduced after transition is called driven series R-L circuit. A series R-L circuit in which an active source is absent after transition is called undriven (or) source free series R-L circuit.

Transient Response of Driven Series R-L circuit:-



$t=0^-$; Switch is open for a long time.

Expression for current through inductor provides the transient Response of the RL series circuit.

Let the initial current through inductor be denoted as I_0. At $t=0^-$ switch is open.

As voltage is not applied to the series R-L circuit, current in the circuit will be zero. But as we already know that current in the inductor doesnot change instantaneously,

$$i(0^-) = I_0 = 0 = i(0^+) \quad \text{——} \; (1)$$

For all $t \geq 0^+$, switch is closed,

Applying KVL,

$$-R\cdot i(t) - L\frac{di(t)}{dt} + V = 0$$

$$R\, i(t) + L\frac{d\,i(t)}{dt} = V$$

Dividing the eq with R,

$$\frac{L}{R}\cdot\frac{d\,i(t)}{dt} = \frac{V}{R} - i(t)$$

by Seperating Variables,

$$\frac{d\,i(t)}{\left[\frac{V}{R} - i(t)\right]} = \frac{R}{L}\cdot dt \quad \rightarrow (2)$$

Integrating both the sides with corresponding variables,

$$-\ln\left[\frac{V}{R} - i(t)\right] = \frac{R}{L}\cdot t + K \rightarrow (3) \quad K - \text{arbitary constant}$$

At $t=0$; $i(t)=0$; $\quad -\ln\left[\frac{V}{R} - 0\right] = \frac{R}{L}\times 0 + K \Rightarrow K = -\ln\left[\frac{V}{R}\right] \rightarrow (4)$

Substitute the value of K,

$$-\ln\left[\frac{V}{R} - i(t)\right] = \frac{R}{L}.t - \ln\left[\frac{V}{R}\right]$$

$$\ln\left[\frac{V}{R} - i(t)\right] - \ln\left[\frac{V}{R}\right] = -\frac{R}{L}.t$$

$$\ln\left[\frac{\frac{V}{R} - i(t)}{\frac{V}{R}}\right] = -\frac{R}{L}.t$$

Applying Antilog,

$$\left[\frac{\frac{V}{R} - i(t)}{\frac{V}{R}}\right] = e^{-\frac{R}{L}.t}$$

$$\Rightarrow \quad i(t) = \frac{V}{R} - \frac{V}{R}.e^{-\frac{R}{L}t} \text{ A} \longrightarrow (5)$$

Above eqn represents the solution of first order non-homogeneous differential Equation obtained by applying KVL to the driven series RL circuit.

Above response is a combination of steady state response (forced response) and transient response (Natural response)

Forced response is denoted by $\frac{V}{R}$ and is due to forcing function i.e., Applied voltage, V.

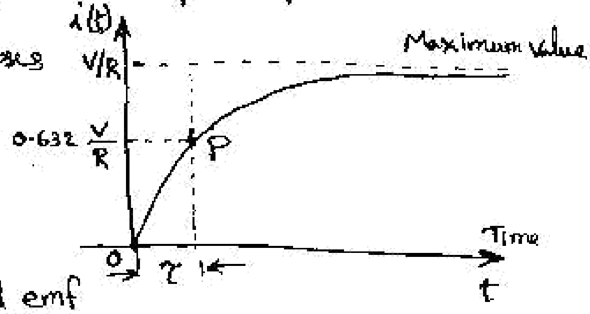
This is also called as the Zero state or steady state response.

Transient Response is denoted by term $\frac{V}{R}.e^{-\frac{R}{L}t}$ in which 't' is involved.

This is a natural response and also called as Zero input response.

From figure, it is clear that, current increases exponentially w.r.t time.

This rising current produces rising flux, which induces emf in the coil



According to lenz's law, the self induced emf opposes the flow of current. Because of this property, current in the coil does not reach it maximum value instantaneously.

From the figure, current rises to P (0.632 times max value) in steady state.

The time required for the current to reach this value is called as time constant of given R-L circuit. It is denoted by τ.

$$\tau = \frac{L}{R} \text{ sec}$$

To determine the significance of τ, substitute $T = \tau, 2\tau, 4\tau, 6\tau \ldots$ in Eq (5)

At $t = \tau$ $\quad i(t) = 0.632 \frac{V}{R}$

$t = 2\tau$ $\quad i(t) = 0.8646 \frac{V}{R}$

$t = 4\tau$ $\quad i(t) = 0.9816 \frac{V}{R}$

$t = 6\tau$ $\quad i(t) = 0.9975 \frac{V}{R}$

It is clear from the above values that current rises to first τ in less time. but after one τ period, this rate slows down for further period of time.
Ideally current reaches to steady state at infinite time, but practically it reaches steady state current value by $t = 6\tau$ (or) 8τ.

Voltage across inductor is

$$V_L(t) = L \frac{di(t)}{dt}$$

by substituting values of $i(t)$ from equation (5)

$$V_L(t) = L \frac{d}{dt}\left[\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right]$$

$$V_L(t) = L\left[\underbrace{\frac{d}{dt}\left(\frac{V}{R}\right)}_{0} - \frac{d}{dt}\left(\frac{V}{R} \cdot e^{-\frac{R}{L}t}\right)\right]$$

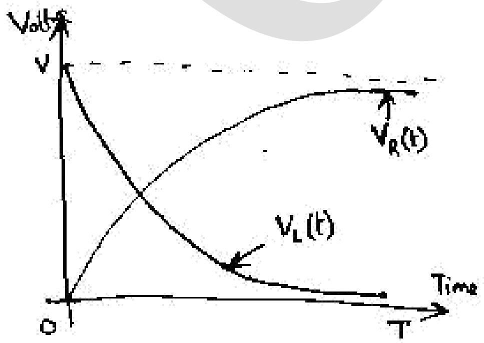
$$V_L(t) = L\left[-\left(\frac{V}{R}\right)\left(\frac{-R}{L}\right) \cdot e^{-\frac{R}{L}t}\right]$$

$$\therefore \boxed{V_L(t) = V \cdot e^{-R/L \cdot t}}$$

Voltage across Resistor is given by,

$$V_R(t) = R \cdot i(t) = R\left[\frac{V}{R} - \frac{V}{R} \cdot e^{-\frac{R}{L}t}\right]$$

$$\Rightarrow V_R(t) = V - V \cdot e^{-\frac{R}{L}t} = V\left(1 - e^{-\frac{R}{L}t}\right) \text{ Volts}$$

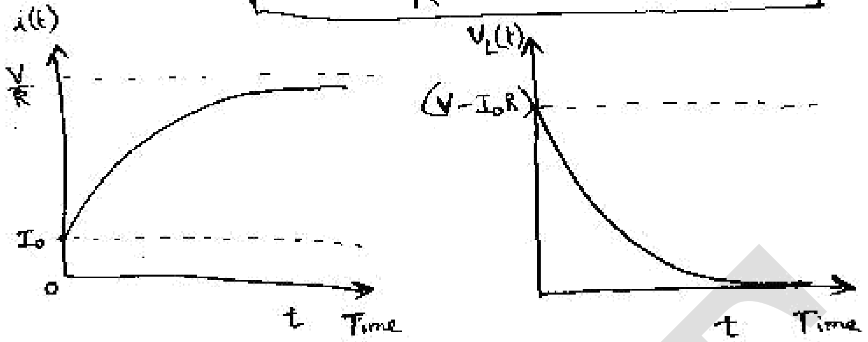


Current through Inductor increases exponentially Voltage across resistor also increases Exponentially but Voltage across inductor decreases exponentially.

When the current reaches it steady state value at infinite time, the voltage across inductor also reaches its steady state value ie, zero. Thus in steady state as voltage across inductor is zero, it acts as a Short circuit.

Considering the series RL circuit, let us assume that inductor carries initial current I_0 before switching action. Then expression of current flowing through inductor is given by

$$i(t) = \frac{V}{R} - \left(\frac{V}{R} - I_0\right) \cdot e^{-\frac{R}{L}t} \quad A$$



Transient Response of i(t) & $V_L(t)$ with inductor carrying some initial current.

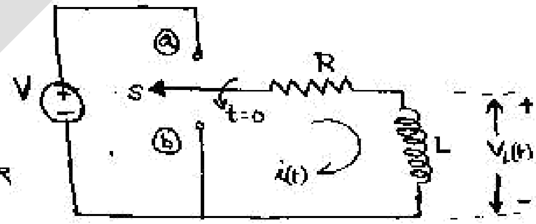
Expression for voltage generated across inductor,

$$V_L(t) = (V - I_0 \cdot R) \cdot e^{-\frac{R}{L}t} \quad V$$

The above response is called Zero-State Response. It is a response to a non-zero input to a circuit with zero initial conditions. Also it is a "Driven circuit."

Transient Response of Source Free (or) Undriven Series R-L circuit:-

Consider a series RL circuit,

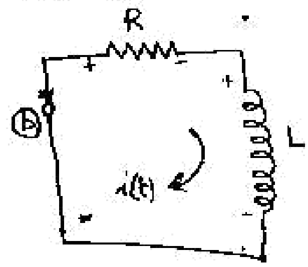


At (a) with switch closed initially for very long time before transition. It indicates that the network before transition is in steady state. So inductor acts as short circuit.

∴ At $t = 0^-$ $\quad i(0^-) = I_0 = \frac{V}{R} = i(0^+)$

For all $t \geq 0^+$ switch is moved to (b) position,

Now the network is without any excitation (or) active source. Hence such a circuit is called source free (or) undriven circuit. As the initial steady state condition is disturbed now, inductor is not shortcircuited in this case.



Applying KVL, $\quad -R \cdot i(t) - L\frac{di(t)}{dt} = 0$

$$i(t) = \frac{-L}{R}\frac{di(t)}{dt}$$

Seperating Variables, $\quad \frac{di(t)}{i(t)} = \frac{-R}{L}dt$

Integrating both the sides wrt corresponding variables,

$$\ln(i(t)) = \frac{-R}{L}.t + K \quad \text{where } K - \text{arbitrary constant.}$$

Find K using initial condition values,

At $t=0$; $i(t) = I_0$

$$\Rightarrow \ln(I_0) = \frac{-R}{L} \times 0 + K \Rightarrow K = \ln[I_0]$$

by substituting K value,

$$\ln(i(t)) = -\frac{R}{L}t + \ln[I_0]$$

$$\Rightarrow \ln[i(t)] - \ln[I_0] = -\frac{R}{L}.t$$

$$\ln\left[\frac{i(t)}{I_0}\right] = -\frac{R}{L}t$$

by Antilog,

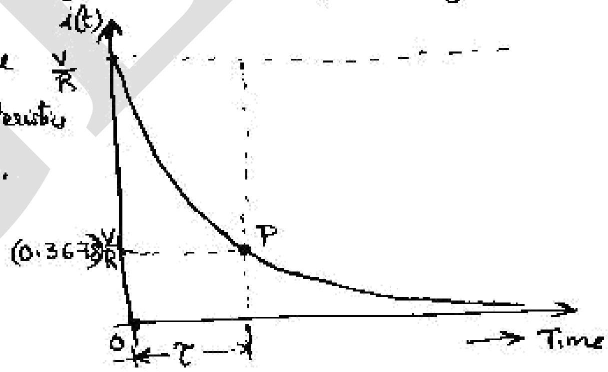
$$\frac{i(t)}{I_0} = e^{-\frac{R}{L}.t}$$

$$\Rightarrow i(t) = I_0 \cdot e^{-\frac{R}{L}.t} \Rightarrow \boxed{i(t) = \frac{V}{R} \cdot e^{-\frac{R}{L}t} \text{ A}}$$

So it is evident that current through inductor exponentially decreases.

At Point P on the graph, the current value $\frac{V}{R}$ is (0.368) times its maximum value. The characteristics of decay is determined by the values of R & L.

$$\because \tau = \frac{L}{R}$$



for $t = \tau$, $i(t) = I_0 \cdot e^{-1} = (0.3678)I_0$

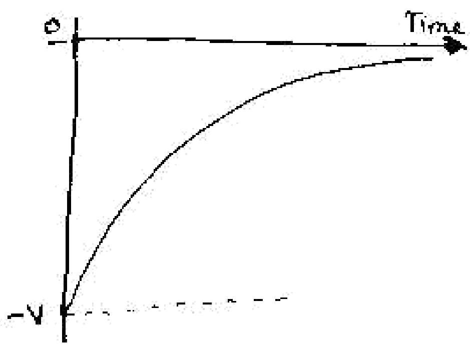
$t = 2\tau$, $i(t) = I_0 \cdot e^{-2} = (0.1353)I_0$

$t = 4\tau$, $i(t) = I_0 \cdot e^{-4} = (0.0183)I_0$

$t = 6\tau$, $i(t) = I_0 \cdot e^{-6} = (0.0024)I_0$

$t = \infty$, $i(t) = I_0 \cdot e^{-\infty} = 0$

For the above values, it is clears that current decreases rapidly to 0.3678 times initial maximum value over first τ, Then rate of decay slows down and reaches steady state at $t = 6\tau$ or 8τ.



$$V_L(t) = L\frac{di}{dt} = L\frac{d}{dt}\left[I_0 \cdot e^{-\frac{R}{L}t}\right] = L \cdot I_0 \cdot \frac{-R}{L} \cdot e^{-\frac{R}{L}t}$$

$$\Rightarrow \boxed{V_L(t) = -I_0 \cdot R \cdot e^{-\frac{R}{L}t}} \; ; \text{ But } I_0 \cdot R = V$$

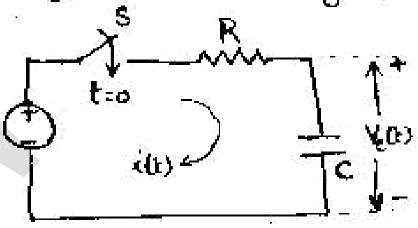
$$\Rightarrow \boxed{V_L(t) = -V \cdot e^{-\frac{R}{L}t} \text{ Volts}}$$

Transient Response of Series RC Circuit for DC Excitation :-

In the Series R C Circuit, capacitor may be initially charged (or) uncharged. As we analyse driven Series RC circuit and undriven (or) Source free Series RC circuit, we obtain the Solution for each case.

Transient Response of Driven Series RC Circuit :-

Consider a Series RC Circuit which is having switch initially open for very long time & at $t=0$ it is closed.



At $t=0^-$ Switch is open.

$$\therefore V_c(0^-) = V_0 = 0 = V_c(0^+) \longrightarrow ①$$

For all $t \geq 0$ Switch S is closed,

Apply KVL,

$$Ri(t) + V_c(t) - V = 0$$

$$Ri(t) + V_c(t) = V \longrightarrow ②$$

$$\because i(t) = C\frac{dV_c(t)}{dt}$$

$$RC\frac{dV_c(t)}{dt} + V_c(t) = V$$

$$RC\frac{dV_c(t)}{dt} = V - V_c(t)$$

Seperating Variables,

$$\frac{dV_c(t)}{(V - V_c(t))} = \frac{1}{RC}dt \longrightarrow ③$$

Integrating both sides w.r.t corresponding Variables,

$$-\ln(V - V_c(t)) = \frac{t}{RC} + K \longrightarrow ④$$

To find K, initial conditions, At $t=0$; $V_c(t) = 0$

$$-\ln(V - 0) = \frac{0}{RC} + K \Rightarrow K = -\ln(V) \longrightarrow ⑤$$

Substituting K,

$$-\ln(V - V_c(t)) = \frac{t}{RC} - \ln(V)$$

$$\ln(V - V_c(t)) - \ln(V) = \frac{-t}{RC}$$

$$\ln\left(\frac{V - V_c(t)}{V}\right) = \frac{-t}{RC}$$

Antilog :-

$$V - V_c(t) = V \cdot e^{-\frac{1}{RC}t} \Rightarrow \boxed{V_c(t) = V - V \cdot e^{-\frac{1}{RC}t} \; V} \longrightarrow ⑥$$

Eq ⑥ is the solution of first order differential equation for driven series RC ckt.

Value of t can be between t = 0 to t = ∞ (Positive values) to obtain $V_c(t)$.

The expression is a combination of steady state response (V) & transient response ($V.e^{-\frac{t}{RC}}$)

$$i(t) = i_c(t) = C \cdot \frac{d(V_c(t))}{dt}$$

$$= C \cdot \frac{d}{dt}\left[V - V.e^{-\frac{t}{RC}}\right]$$

$$i(t) = C\left[0 - V \cdot \frac{-1}{RC} \cdot e^{-\frac{t}{RC}}\right]$$

$$\boxed{i(t) = \frac{V}{R} \cdot e^{-\frac{t}{RC}}\ A} \longrightarrow ⑦$$

$\tau = RC$ sec

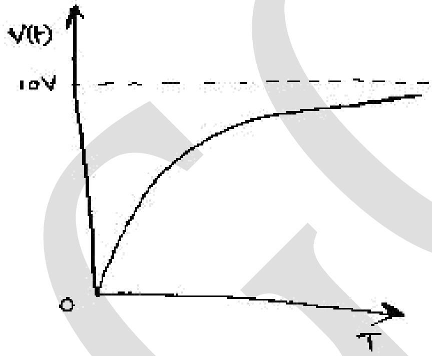
For $t = \tau$; $V_c(t) = V - V \cdot e^{-1} = 0.632V$

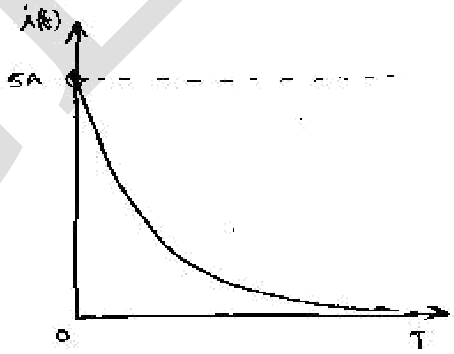
$t = 2\tau$; $V_c(t) = V - V \cdot e^{-2} = 0.8646V$

$t = 4\tau$; $V_c(t) = V - V \cdot e^{-4} = 0.9816V$

$t = 6\tau$; $V_c(t) = V - V \cdot e^{-6} = 0.9975V$

From the above values, it is clear that at $t = \tau$, voltage across capacitor rapidly rises to 0.632 times steady state value, then rate of increase slows down.



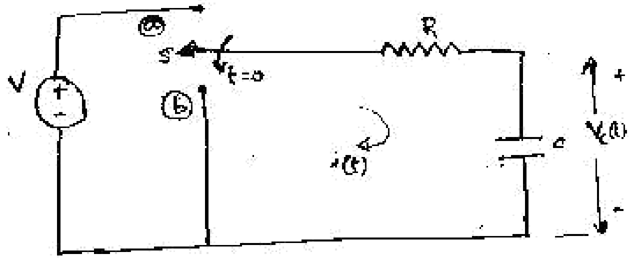


Transient Response of Source Free (or) Undriven Series RC Circuit:-

Initially at $t=0^-$ Switch is in ⓐ position

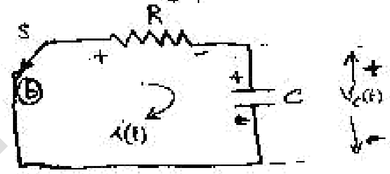
At $t=0$; Switch is moved to ⓑ position

Here we will find the discharge of Capacitor through resistor expressed as Voltage across capacitor as a function of time(t)



∴ At $t=0^-$ Switch is in position ⓐ Network in steady state & C → O.C

$$\therefore V_c(0^-) = V_0 = V = V_c(0^+) \longrightarrow ①$$



For All $t \geq 0$ Switch is moved to position ⓑ

Now Network is without excitation and called as Source free (or) Undriven series RC circuit. Now Voltage across capacitor varies exponentially. steady state is disturbed.

By applying KVL,

$$i(t)\cdot R + V_c(t) = 0 \longrightarrow ②$$

$$RC \cdot \frac{dV_c(t)}{dt} = -V_c(t) \qquad \because i(t) = C\frac{dV_c(t)}{dt}$$

Seperating variables,

$$\frac{dV_c(t)}{V_c(t)} = \frac{-1}{RC}\,dt$$

Integrating on both sides with respect to corresponding variables

$$\ln[V_c(t)] = \frac{-t}{RC} + K \longrightarrow ③$$

find K; At $t=0$; $V_c(t) = V_0 = V$

$$\ln[V_0] = \frac{-0}{RC} + K \Rightarrow K = \ln[V_0] \longrightarrow ④$$

Substituting K in ③,

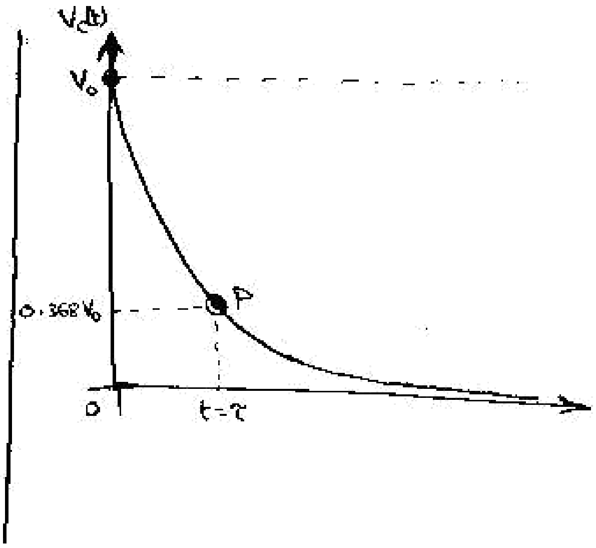
$$\ln[V_c(t)] = \frac{-t}{RC} + \ln[V_0]$$

$$\ln[V_c(t)] - \ln[V_0] = \frac{-t}{RC}$$

$$\ln\left[\frac{V_c(t)}{V_0}\right] = \frac{-t}{RC}$$

Take Antilog,

$$\boxed{V_c(t) = V_0 \cdot e^{-\frac{t}{RC}} \text{ Volts}}$$



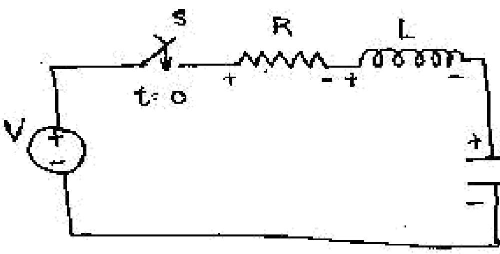
For $t=\tau$; $V_c(t) = 0.368\,V_0$ | $t=4\tau$; $V_c(t) = 0.0183\,V_0$

$=2\tau$; $V_c(t) = 0.1353\,V_0$ | 6τ; $V_c(t) = 0.0024\,V_0$

Transient Response of Series R-L-C Circuit for DC Excitation:-

In Series RLC circuit as two energy storing elements are present, when we apply KVL, differential equation of Second order can be obtained.

Consider a Series RLC circuit,



At $t=0^-$, Switch is open.

$$i(0^-) = 0 = i(0^+) \quad \& \rightarrow ①$$

$$V_c(0^-) = 0 = V_c(0^+) \quad \rightarrow ②$$

For all $t \geq 0^+$ switch is closed.

Applying KVL,

$$i(t)\cdot R + L\cdot\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = V \longrightarrow ③$$

Splitting the limits,

$$i(t)\cdot R + L\,\frac{di(t)}{dt} + \underbrace{\frac{1}{C}\int_{-\infty}^{0^-} i(t)\,dt}_{\text{Initial voltage (zero)}} + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt = V$$

$$i(t)\cdot R + L\,\frac{di(t)}{dt} + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt = V \longrightarrow \text{Integrodifferential equation}$$

differenciating both sides w.r.t t,

$$L\,\frac{d^2 i(t)}{dt^2} + R\,\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

$$\Rightarrow \frac{d^2 i(t)}{dt^2} + \frac{R}{L}\,\frac{di(t)}{dt} + \frac{1}{LC}\,i(t) = 0 \longrightarrow ④$$

Eq ④ Indicates second order differential equation for which solution can be obtained by getting characteristic (or) auxillary equation by replacing $\frac{d}{dt}$ with s & $\frac{d^2}{dt^2}$ with s^2.

$$\Rightarrow s^2 i(t) + \frac{R}{L}s\,i(t) + \frac{1}{LC}\,i(t) = 0$$

The response of circuit depends on the nature of roots of the auxillary equation

The two roots are

$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

The following quantities are necessary to determine the response according to nature of roots.

① <u>Critical Resistance (R_{cr})</u> :- This value of resistance which reduces squareroot term to zero, giving real, equal and negative roots.

$$\frac{R_{cr}}{2L} = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{R_{cr} = 2\sqrt{\frac{L}{C}}}$$

② <u>Damping Ratio (ξ)</u> ; This ratio is the indication of the opposition from the circuit to cause oscillations in its response. More the value of this ratio, less the chances of oscillation in the response. It is the ratio of actual Resistance in the circuit to the critical resistance. denoted by greek letter Zeta (ξ)

$$\boxed{\xi = \frac{R}{R_{cr}} = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

③ <u>Natural frequency (ω_n)</u> : If the damping is made zero then the response oscillates with natural frequency without any opposition. Such a frequency when $\xi = 0$ is called natural frequency of oscillations,

$$\boxed{\omega_n = \frac{1}{\sqrt{LC}}}$$

Using these values, the root of the equation are

$$S_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$$

Thus the response is totally dependent on the values of ξ.

Let $\alpha = \xi\omega_n$ and $\omega_d = \sqrt{1-\xi^2}$

$\omega_d \rightarrow$ actual freq of oscillations i.e. damped frequency when $\xi = 0$ we get $\omega_d = \omega_n$ (natural freq)

General solution,

$$\boxed{i(t) = K_1 e^{(-\alpha + j\omega_d)t} + K_2 e^{(-\alpha - j\omega_d)t}}$$

when $0 < \xi < 1$, Imaginary term $j\omega_d$ exists and we get sine & cosine terms in response as $e^{j\theta} = \cos\theta + j\sin\theta$

Such a network is called <u>underdamped Network</u>. Roots of characteristic equation are complex conjugates with negative real parts. Due to -ve real part oscillations are damped and vanish after some time.

when $\xi = 1$, The roots are real, equal and negative. Here Response is exponential and fastest if

The response of such case takes form,

$$i(t) = K_1 e^{s_1 \cdot t} + K_2 \cdot e^{s_2 \cdot t}$$

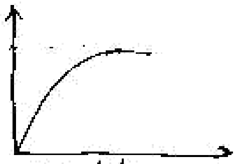
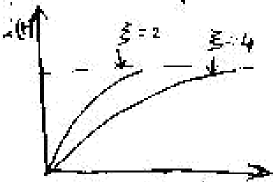
when $\xi > 1$, damping becomes high and the response remains exponential but becomes more and more sluggish and slow as ξ increases. Such cases are called <u>overdamped</u>.

The response takes the form,

$$i(t) = K_1 \cdot e^{s_1 \cdot t} + K_2 \cdot e^{s_2 \cdot t}$$

when $\xi = 0$, the damping becomes zero and response oscillates with maximum frequency ω_n. Such case is called <u>undamped</u> case. The output is oscillations with constant frequency and amplitude i.e., sustained oscillation.

<u>Tabulated Responses for each case</u>

Range of ξ	Nature of roots	Form of Response	Circuit Classification	Nature of Response
$\xi = 0$	Purely Imaginary $\pm j\omega_0$	$K_1 \cos\omega t + K_2 \sin\omega t$	undamped	 Sustained oscillations
$0 < \xi < 1$	Complex conjugates with negative real part $-\alpha \pm j\omega d$	$K_1 e^{-\alpha t} \cos\omega t + K_2 e^{-\alpha t} \sin\omega t$	underdamped	 Damped oscillations
$\xi = 1$	Real equal Negative $-\alpha, -\alpha$	$K_1 e^{-\alpha t} + K_2 \cdot e^{-\alpha t}$	Critically damped	 Exponential & critical response
$1 < \xi < \infty$	Real unequal negative	$K_1 \cdot e^{s_1 t} + K_2 e^{-s_2 \cdot t}$	Overdamped	 Exponential & slow

Specifications from step Response of Second order circuit :-

Consider an second order system which is underdamped ($\xi > 1$), is excited by unit step input.

Now a Transient o/p is damped oscillatory and finally system tries to achieve steady state almost equal to unity i.e, magnitude of the step applied.

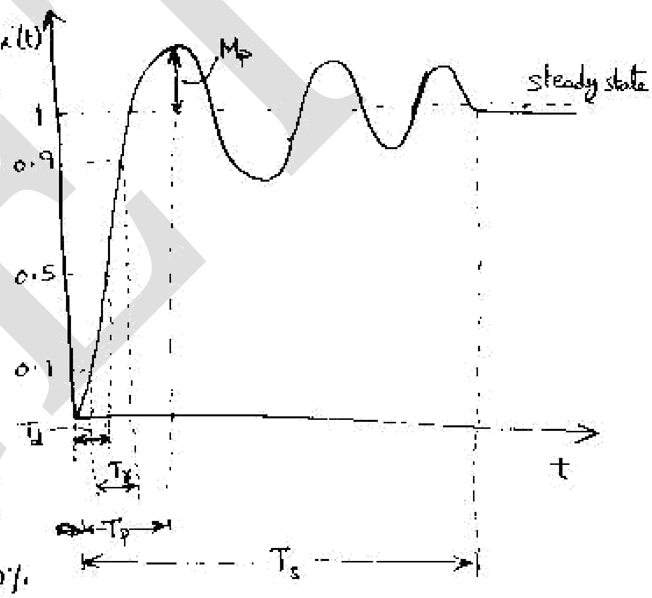
In terms of ξ and ω_n, the eqution for response is

$$i(t) = i_{ss} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{where } \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$i_{ss} \rightarrow$ steady state response which remains at $t \rightarrow \infty$

The remaining part is transient part which dies out after some time.

Such a response is given as



① Delay Time (T_d) :- It is the time required by the response to reach 50% of its steady state value, in first attempt.

$$T_d = \frac{1 + 0.7\xi}{\omega_n} \text{ Seconds}$$

② Rise Time (T_r) :- It is the time required by the response to rise from 10% to 90% of the final value

$$T_r = \frac{\pi - \theta}{\omega_d} \text{ Seconds.}$$

③ Peak Time (T_p) :- At the time of first overshoot response achieves a peak. The time at which first peak overshoot occurs is called Peak Time.

$$T_p = \frac{\pi}{\omega_d} \text{ Seconds}$$

④ Peak overshoot (M_p) :- The amount by which the response overshoots its final value, during the Peak overshoot is called magnitude of peak overshoot.

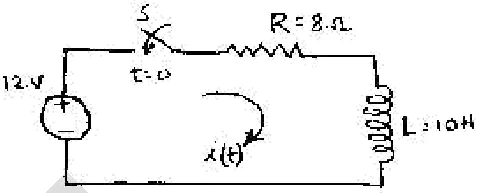
$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

⑤ Settling time (T_s) :- The time required for the response to decrease and becomes steady state at its steady state value and remains thereafter within ± 2% of its final value is Setting time.

$$T_s = \frac{4}{\xi\omega_n} \text{ Seconds}$$

Problems (For RL circuit)

① The circuit shown in the figure, initially switch is kept open for long time. At $t=0$, switch K is closed. obtain expression for current in the circuit for $t>0$. find value of current at $t = 0.25$ sec. What will be the current in the circuit in one time constant period? Determine the instant of time at which the current in the circuit reaches to 1.2A.



Sol:- At $t=0^-$ switch S is open.

$$i(0^-) = 0 = I_0 = i(0^+)$$

For all $t \geq 0^+$, switch S is closed.

By applying KVL,

$$+8\, i(t) + 10 \frac{di(t)}{dt} = 12$$

$$\frac{di(t)}{dt} + 0.8\, i(t) = 1.2$$

This equation is first order Non-homogeneous differential Equation of type $\frac{di}{dt} + Pi = Q$

where $P = 0.8$ & $Q = 1.2$;

The Solution for such an equation is given by

$$i(t) = e^{-Pt} \int_0^t Q \cdot e^{Pt} dt + K \cdot e^{-Pt}$$

Substituting P & Q in above equation,

$$i(t) = e^{-0.8t} \int_0^t 1.2 \cdot e^{0.8t} dt + K \cdot e^{-0.8t}$$

$$= 1.2\, e^{-0.8t} \left[\frac{e^{0.8t}}{0.8} \right]_0^t + K \cdot e^{-0.8t}$$

$$= 1.5\, e^{-0.8t} (e^{0.8t} - e^{0}) + K \cdot e^{-0.8t}$$

$$\Rightarrow i(t) = 1.5 (1 - e^{-0.8t}) + K \cdot e^{-0.8t} \longrightarrow ①$$

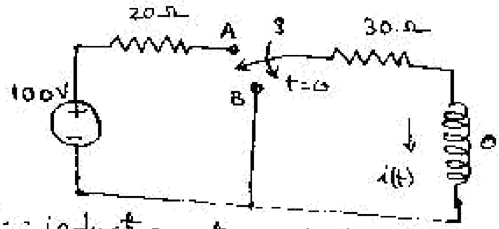
To find K, at $t=0$; $i(t) = 0$

$$\Rightarrow 0 = 1.5 (1 - e^{-0}) + K \cdot e^{-0} \Rightarrow K = 0$$

Hence $i(t) = 1.5 (1 - e^{-0.8t})$ A

- At $t = 0.25$ sec; $i(t) = 1.5 (1 - e^{-(0.8)(0.25)}) = 0.2719$ A $\longrightarrow$ ②
- For $\tau = \frac{L}{R} = \frac{10}{8} = 1.25$ sec; $\Rightarrow$ At $t = \tau$, $i(t) = 1.5 (1 - e^{-(0.8)(1.25)}) = 0.9481$A $\longrightarrow$ ③
- At $i(t) = 1.2$A; $1.2 = 1.5 (1 - e^{-0.8t}) \Rightarrow t = 2.01$ sec $\longrightarrow$ ④

② In the circuit shown, initially switch S is kept at position A for long time. At t=0, switch is moved to position B. Find expression for current for t>0. Find value of current at t = 6.6667 msec, 13.3334 msec, 20 msec. Plot the variation of current through inductor versus time.

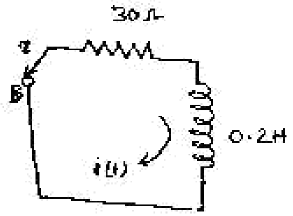


Sol:- At $t = 0^-$ switch S is in position A.

$\therefore i(0^-) = I_0 = \frac{100}{30+20} = 2A = i(0^+)$ $\because$ inductor acts as short circuit

For all $t \geq 0^+$ switch S is moved to position B

Hence network becomes undriven series RL circuit.



Apply KVL:

$$30\, i(t) + 0.2 \frac{d\,i(t)}{dt} = 0$$

$$0.2 \frac{di(t)}{dt} + 30\, i(t) = 0$$

This equation is in the form of $a_0 \frac{di(t)}{dt} + a_1 i(t) = 0$ (first order homogeneous diff eq)

where $a_0 = 0.2$; $a_1 = 30$

Solution for such equation is

$$i(t) = K \cdot e^{-\frac{a_1}{a_0} \cdot t}$$

Substituting a_0 & a_1,

$$i(t) = K \cdot e^{-\frac{30}{0.2}t} = K \cdot e^{-150t} \longrightarrow ①$$

find K, at t=0; i(t) = 2A;

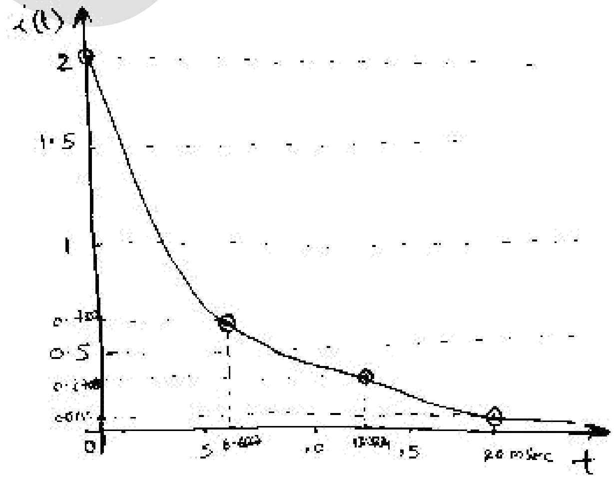
$$2 = K \cdot e^{-150\times 0} \Rightarrow K = 2$$

$$\Rightarrow i(t) = 2 \cdot e^{-150t}$$

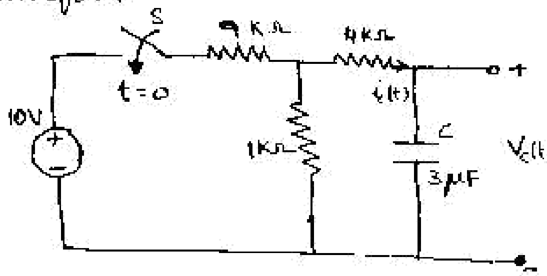
Now At t = 6.6667 msec ; $i(t) = 2 \cdot e^{-150(6.6667\times 10^{-3})} = 0.7357$ A

At t = 13.3334 msec ; $i(t) = 2 \cdot e^{-150(13.3334\times 10^{-3})} = 0.2706$ A

At t = 20 msec ; $i(t) = 2 \cdot e^{-150(20\times 10^{-3})} = 0.0995$ A



① In the circuit shown in figure, 10V battery is connected to the network by closing switch at $t=0$. Assume that initial voltage on capacitor is zero. Determine expression for $V_c(t)$ and $i_c(t)$ and sketch the waveform.



Sol: At $t=0^-$, Switch is open.

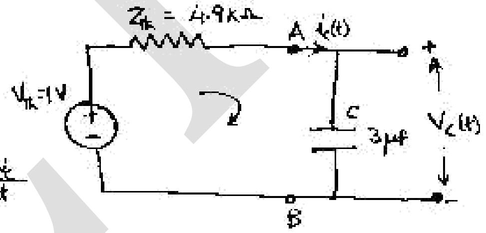
$$V_c(0^-) = 0 = V_0 = V_c(0^+)$$

For all $t \geq 0^+$, Switch is closed.

First Reduce the network to thevinens equivalent & apply KVL.

$$V_{th} = V_{oc} = 10\left[\frac{1K}{9K+1K}\right] = 1V$$

$$Z_{th} = (9K \parallel 1K) + 4K = \frac{(9K \times 1K)}{(9K+1K)} + 4K = 4.9K\Omega$$



Now Apply KVL;

$$(4.9\times10^{3})\, i_c(t) + V_c = 1 \qquad i_c = C\cdot\frac{dV_c}{dt}$$

$$\Rightarrow (4.9\times10^{3})(3\times10^{-6})\frac{dV_c}{dt} + V_c = 1$$

$$\frac{dV_c}{dt} + 68.027V_c = 68.027 \qquad \left[\text{In the form } \frac{dV_c}{dt} + PV_c = Q\right]$$

Solution for above eq

$$V_c = e^{-Pt}\int_0^t 68.027 \times e^{Pt}\,dt + K\cdot e^{-Pt}$$

$$= e^{-68.027t}\int_0^t 68.027\cdot e^{68.027t} + K\cdot e^{-68.027t}$$

$$= e^{-68.027t}\left[\frac{e^{68.027}}{68.027}\right]_0^t \times 68.027 + K\cdot e^{-68.027t}$$

$$= e^{-68.027t}\left[e^{68.027t} - 1\right] + K\cdot e^{-68.027t}$$

$$\Rightarrow V_c = 1 - e^{-68.027t} + K\cdot e^{-68.027t}$$

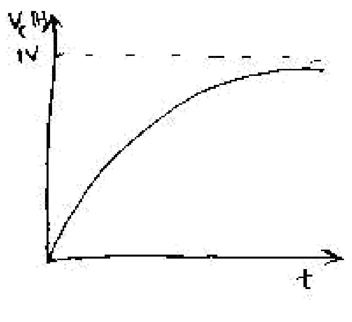
Find K, at $t=0$; $V_c = 0$

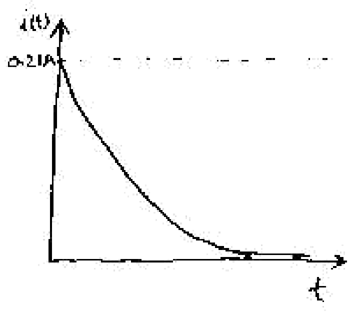
$$0 = 1 - e^0 + K\cdot e^0 \Rightarrow K = 0$$

$$\Rightarrow V_c = 1 - e^{-68.027t} \text{ Volts}$$

$$i_c = (3\times10^{-6})\frac{d}{dt}\left(1 - e^{-68.027t}\right)$$

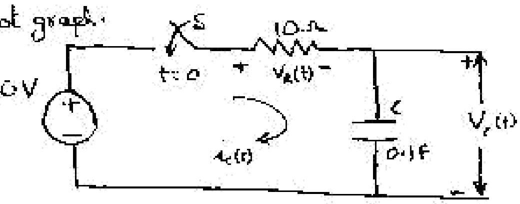
$$\Rightarrow i_c = 0.21\cdot e^{-68.027t}\ \text{mA}$$





Assignment Problem

① A Series RC circuit shown in figure consists of R=10Ω and C=0.1F. Initially switch is kept open for very long time. At t=0, it is closed. Find expression for $V_c(t)$, $i_c(t)$ and $V_R(t)$ against time and plot graph.



$V_c(t) = 20(1-e^{-t})$ V

$i_c(t) = 2e^{-t}$ A

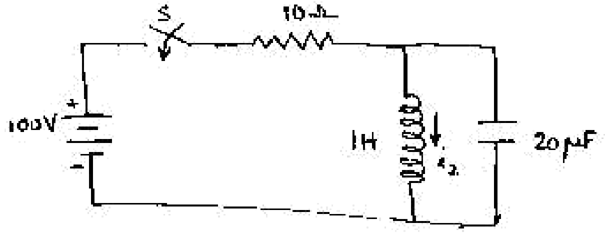
$V_R(t) = 20 \cdot e^{-t}$ V

RLC

②

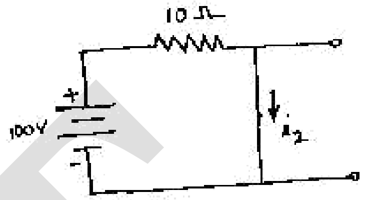
Problems (for RLC circuits)

① In the network shown in figure, switch S is closed and a steady state is reached in the network. At $t=0$, the switch is opened. Find an expression for the current in the inductor, $i_2(t)$.



At $t=0^-$, switch is closed & network is in steady state.

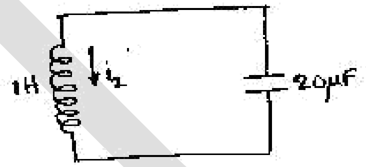
Inductor acts as short circuit while capacitor acts as open circuit.



$$i_2(0^-) = I_0 = \frac{V}{R} = \frac{100}{10} = 10\text{A} = i_2(0^+)$$

$$V_c(0^-) = 0 = V_c(0^+)$$

For all $t \geq 0^+$, switch is opened.



Applying KVL to a closed path,

$$1\frac{di_2}{dt} + \frac{1}{20\times10^{-6}}\int_{-\infty}^{t} i_2\,dt = 0$$

$$\frac{di_2}{dt} + \underbrace{\frac{1}{20\times10^{-6}}\int_{-\infty}^{0} i_2\,dt}_{0} + \frac{1}{20\times10^{-6}}\int_{0}^{t} i_2\,dt = 0$$

$$\Rightarrow \frac{di_2}{dt} + \frac{1}{20\times10^{-6}}\int_{0}^{t} i_2\,dt = 0$$

Differentiating w.r.t t,

$$\frac{d^2 i_2}{dt^2} + \frac{1}{20\times10^{-6}} i_2 = 0$$

Let $s = \frac{d}{dt}$, Now $s^2 i_2 + (50\times10^3)\, i_2 = 0 \longrightarrow ①$

Roots of the above equation are

$$s_{1,2} = \frac{-0 \pm \sqrt{0 - 4(1)(50\times10^3)}}{2(1)} = \frac{\pm\sqrt{-(200\times10^3)}}{2}$$

$s_{1,2} = \pm j(223.6)$ [The roots of eq are equal & imaginary

Solution of eq ① is

$$i_2(t) = K_1\cdot e^{s_1 t} + K_2\cdot e^{s_2 t}$$

$$\therefore i_2(t) = K_1 e^{+j(223.6)t} + K_2\cdot e^{-j(223.6)t}$$

$$i_2(t) = K_1\left[\cos(223.6)t + j\sin(223.6)t\right] + K_2\left[\cos(223.6)t - j\sin(223.6)t\right]$$

$$i_2(t) = (K_1+K_2)\cos(223.6)t + j(K_1-K_2)\sin(223.6)t$$

Let $(K_1+K_2) = K_3$ and $(K_1-K_2) = K_4$

$$\therefore i_2(t) = K_3\cos(223.6)t + jK_4\sin(223.6)t$$

Find K_3 & K_4,

at $t=0$; $i_2 = 10 = i_2(0)$ &

$$\frac{di_2}{dt}(0) + 0 = 0 \Rightarrow \frac{di_2}{dt}(0) = 0$$

Substituting $t=0$,

$$i_2(0) = K_3 \cos(0) + K_4 \sin(0)$$

$$10 = K_3(1) + K_4(0)$$

$$K_3 = 10$$

differenciating eq wrt t

$$\frac{di_2}{dt} = K_3[-\sin(223.6)t] + K_4[\cos(223.6)]$$

~~$i_2(0) = K_3 \cos(0) + K_4 \sin(0)$~~

At $t=0$,

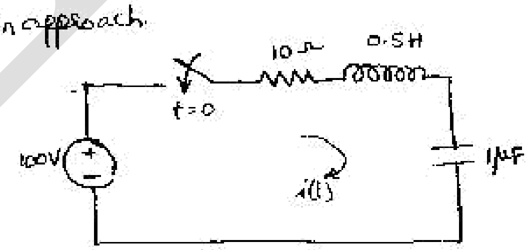
$$\frac{di_2}{dt}(0) = 0 = K_3[-\sin(0)] + K_4[\cos(0)]$$

$$K_4 = 0$$

Substituting values of K_3 & K_4,

$$\boxed{i_2(t) = 10 \cos(223.6)t \ \text{A}}$$

② obtain current $i(t)$ for $t \geq 0$ using time domain approach.



Sol: At $t=0^-$, Switch is open.

$$i_L(0^-) = 0 = i_L(0^+)$$

$$V_c(0^-) = 0 = V_c(0^+)$$

for all $t \geq 0^+$, Switch is closed.

Apply KVL,

$$10\, i(t) + 0.5 \frac{di(t)}{dt} + \frac{1}{1\times10^{-6}} \int_{-\infty}^{t} i(t)\, dt = 100$$

$$10\, i(t) + 0.5 \frac{di(t)}{dt} + \underbrace{\frac{1}{1\times10^{-6}} \int_{-\infty}^{0} i(t)\, dt}_{0} + \frac{1}{1\times10^{-6}} \int_{0^-}^{t} i(t)\, dt = 100$$

$$10\, i(t) + 0.5 \frac{di(t)}{dt} + \frac{1}{1\times10^{-6}} \int_{0}^{t} i(t)\, dt = 100$$

differenciating eq wrt t,

$$10 \frac{di(t)}{dt} + 0.5 \frac{d^2 i(t)}{dt} + \frac{i(t)}{1\times10^{-6}} = 0$$

divide with 0.5 on bothsides,

$$\frac{d^2 i(t)}{dt} + 20 \frac{di(t)}{dt} + 2\times10^6 = 0 \Rightarrow S^2 + 20S + 2\times10^6 = 0$$

roots of auxiliary eq can be found by,

$$S_{1,2} = \frac{-20 \pm \sqrt{(20)^2 - 4(1)(2\times10^6)}}{2(1)} = \frac{-20 \pm j2828.36}{2}$$

$$S_{1,2} = -10 \pm 1414.18 \implies S_1 = -\alpha + j\omega_d = -10 + j1414.18$$

$$S_2 = -\alpha - j\omega_d = -10 - j1414.18$$

So roots are complex conjugate with -ve real parts

$$i(t) = K_1 \cdot e^{-\alpha t} \cos\omega_d t + K_2 \cdot e^{-\alpha t} \cdot \sin\omega_d t$$

$$= K_1 e^{-10t} \cos(1414.18)t + K_2 \cdot e^{-10t} \cdot \sin(1414.18)t$$

At $t=0$; $i(t) = 0$,

$$\implies 0 = K_1 \cdot e^0 \cos(0) + K_2 \cdot e^0 \sin(0)$$

$$\implies K_1 = 0$$

Now Equation becomes

$$i(t) = K_2 \cdot e^{-10t} \cdot \sin(1414.18)t$$

differenciating w.r.t t,

$$\frac{di(t)}{dt} = K_2 \left[e^{-10t} \cdot (1414.18) \cos 1414.18t + \sin 1414.18t \, (-10) \, e^{-10t} \right]$$

$$\therefore \frac{di(t)}{dt} = K_2 \cdot e^{-10} \left(1414.18 \times \cos 1414.18t - 10 \sin 1414.18t \right)$$

At $t=0$,

$$\frac{di(0)}{dt} = K_2 \cdot e^{-0} \left(1414.18 \times \cos 0 - 10 \sin 0 \right) = K_2 (1414.18)$$

Now the Integrodifferential becomes

$$10\, i(0) + 0.5 \frac{di(0)}{dt} + \frac{1}{1\times10^{-6}} \int_0^0 i(0)\,dt = 100$$

$$0 + 0.5 \frac{di}{dt}(0^+) + 0 = 100$$

$$\frac{di}{dt}(0^+) = 200$$

Equating

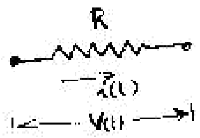
$$K_2 (1414.18) = 200$$

$$K_2 = 0.1414$$

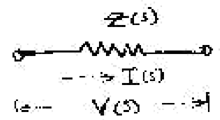
$$\implies i(t) = 0.1414 \cdot e^{-10t} \cdot \sin 1414.18t \text{ A}$$

Transform of Basic R, L & C components :-

① **Resistor :-**



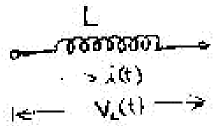
$V(t) = i(t) \times R$



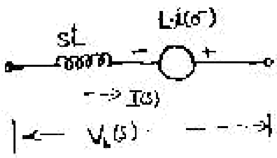
$Z(s) = \frac{V(s)}{I(s)}$

$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} = G$

② **Inductor :-**



$V_L(t) = L \cdot \frac{di(t)}{dt}$



$V_L(s) = L\left[s \cdot I(s) - i(0^-)\right]$

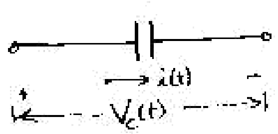
$\therefore V_L(s) = s \cdot L \cdot I(s) - L \cdot i(0)$ ← Voltage source

$\Rightarrow V_L(s) = s \cdot L \cdot I(s)$

$Z(s) = \frac{V_L(s)}{I(s)} = sL$

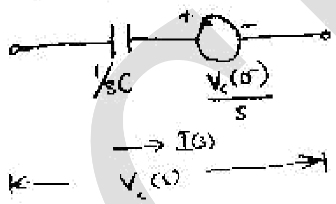
$Y(s) = \frac{1}{Z(s)} = \frac{1}{sL}$

③ **Capacitor :-**



$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = \underbrace{\frac{1}{C}\int_{-\infty}^{0^-} i(t)\,dt}_{\text{Initial Voltage}} + \frac{1}{C}\int_{0}^{t} i(t)\,dt = V_c(0^-) + \frac{1}{C}\int_{0}^{t} i(t)\,dt$

Taking Laplace Transform



$V_c(s) = \frac{1}{C}\left[\frac{I(s)}{s}\right] + \frac{V_c(0^-)}{s}$ ← Voltage source in L.T

$V_c(s) = \frac{I(s)}{sC}$; $Z(s) = \frac{V_c(s)}{I(s)} = \frac{1}{sC}$

$Y(s) = \frac{1}{Z(s)} = sC$

Advantages of S-Domain Network :-

① It is simple to obtain the corresponding transform Impedance by replacing each element

② All elements behave as Impedances in S domain, so using various simplification techniques, the impedances can be combined easily to obtain simple form of Network.

③ The terms related to voltage drops across the elements are of simple form like [I(s) × Z(s)]

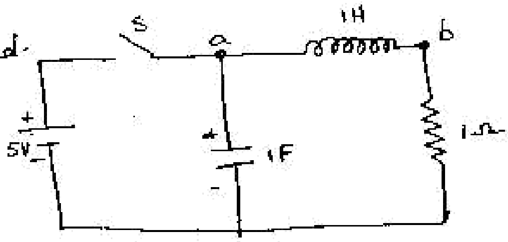
④ No Integral (or) differential terms are present in the set of network equations.

⑤ From s domain network, the system function & resultant transformer impedance can be easily obtained. These concepts are important to analyze the network in S domain.

Note :- The total equivalent impedance of the S domain network, obtained as viewed through the input terminals is called driving point Impedance (Z(s)) of the network. Such Network is called as Laplace domain Network.

Laplace Transform Problems:-

① In the figure, the switch is initially closed. After steady state the switch is opened. Determine the nodal voltages $V_a(t)$ & $V_b(t)$ using L.T.

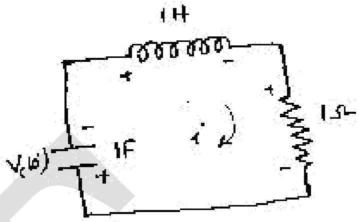


Sol: when switch is closed

$$V_c = 5V \; ; \; \& \; i = \frac{5}{1} = 5A$$

At $t=0$, Switch is opened

$$\therefore V_c(0^-) = 5V \quad \& \quad i(0^-) = 5A$$



Applying KVL,

$$L\frac{di(t)}{dt} + i(t)\cdot R + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 0.$$

$$\frac{di(t)}{dt} + i(t) + \int_{-\infty}^{0} i(t)\,dt + \int_{0^-}^{t} i(t)\,dt = 0$$

$V(0^-) = 5V$ but $-ve$

$$\therefore \frac{di(t)}{dt} + i(t) - 5 + \int_{0}^{t} i(t)\,dt = 0$$

Taking Laplace Transform,

$$[s\cdot I(s) - i(0^-)] + I(s) + \frac{I(s)}{s} = \frac{5}{s}$$

$$I(s)\left[s+1+\frac{1}{s}\right] = \frac{5}{s} + 5$$

$$I(s) = \frac{5(s+1)}{(s^2+s+1)} = 5\left[\frac{s+1}{s^2+s+\frac{1}{4}+1-\frac{1}{4}}\right] = 5\left[\frac{s+1}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right] = 5\left[\frac{s+\frac{1}{2}+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

$$\Rightarrow I(s) = 5\left[\frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} + \frac{1}{2}\times\frac{2}{\sqrt{3}}\,\frac{(\frac{\sqrt{3}}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right] = 5\left[\frac{(s+a)}{(s+a)^2+(\omega)^2} + K\times\frac{\omega}{(s+a)^2+\omega^2}\right]$$

$$\therefore i(t) = L^{-1}[I(s)] = 5\left[e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{-\frac{1}{2}t}\cdot\sin\frac{\sqrt{3}}{2}t\right] = 5e^{-0.5t}\left[\cos(\tfrac{\sqrt{3}}{2}t)+0.5773\sin(\tfrac{\sqrt{3}}{2}t)\right]$$

$$V_b(t) = i(t)\times R = 5.e^{-0.5t}\left[\cos(\tfrac{\sqrt{3}}{2}t) + 0.57738\sin(\tfrac{\sqrt{3}}{2}t)\right] V$$

$$V_a(t) = \text{Drop across } L + V_b(t) = L\frac{di(t)}{dt} + V_b(t)$$

$$\Rightarrow V_a(s) = 1\times[s\cdot I(s) - i(0^-)] + V_b(s) = s\cdot I(s) - 5 + I(s) = (s+1)I_s - 5$$

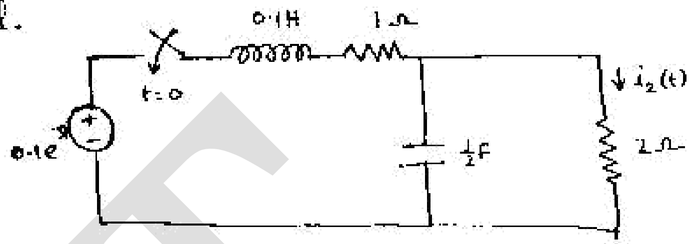
$$\Rightarrow V_a(s) = \frac{(s+1)\times 5(s+1)}{(s^2+s+1)} - 5 = 5\left[\frac{s^2+2s+1-s^2-s-1}{s^2+s+1}\right]$$

$$\Rightarrow V_a(s) = 5\left[\frac{s}{s^2+s+1}\right] = 5\left[\frac{s+\frac{1}{2}-\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

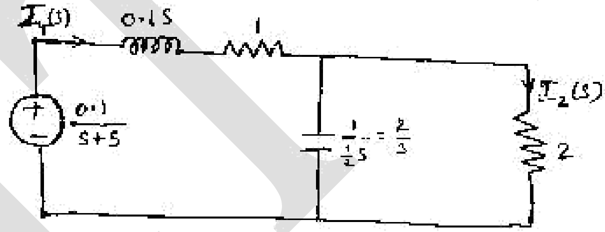
$$\Rightarrow V_a(s) = 5\left[\frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} - \frac{1}{2}\times\frac{2}{\sqrt{3}}\,\frac{(\frac{\sqrt{3}}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

$$\therefore V_a(t) = L^{-1}[V_a(s)] = 5\cdot e^{-0.5t}\left[\cos(\tfrac{\sqrt{3}}{2}t) - 0.5773\sin(\tfrac{\sqrt{3}}{2}t)\right]\ V$$

② find $i_2(t)$ by laplace transform method.



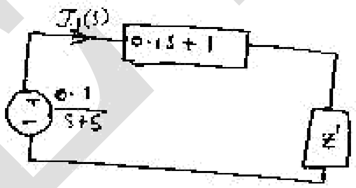
∵ Initial conditions are zero,



$$Z' = \left(\frac{2}{s}\right)\|\,2 = \frac{\frac{2}{s}\times 2}{\frac{2}{s}+2} = \frac{4}{2+2s} = \frac{2}{(s+1)}$$

$$I_T(s) = \frac{V}{Rz} = \frac{\frac{0.1}{s+5}}{1+0.1s+\frac{2}{(s+1)}} = \frac{0.1(s+1)}{(s+5)(s^2+11s+30)}$$

$$\Rightarrow I_T(s) = \frac{(s+1)}{(s+5)^2(s+6)}$$



we need $I_2(s)$

$$\therefore I_2(s) = I_T(s)\left[\frac{(\frac{2}{s})}{\frac{2}{s}+2}\right] = \frac{I_T(s)}{s+1}$$

$$= \frac{(s+1)}{(s+1)(s+5)^2(s+6)} = \frac{1}{(s+5)^2(s+6)} = \frac{A}{(s+5)^2}+\frac{B}{(s+5)}+\frac{C}{(s+6)}$$

Partial fraction

$$\Rightarrow A(s+6)+B(s+5)(s+6)+C(s+5)^2 = 1$$

$$\Rightarrow s^2[B+C] + s[A+11B+10C] + [6A+30B+25C] = 1$$

$$\Rightarrow B+C=0;\quad A+11B+10C=0;\quad 6A+30B+25C=1$$

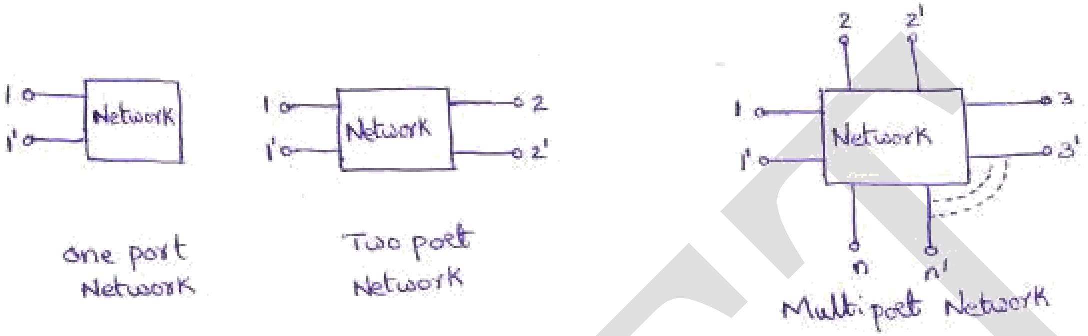
But $C = \frac{1}{(s+5)^2}\Big|_{s=-6} = 1 \Rightarrow B=-1;\ A=1$

$$\therefore I_2(s) = \frac{1}{(s+5)^2} - \frac{1}{(s+5)} + \frac{1}{(s+6)}$$

$$i_2(t) = L^{-1}[I_2(s)] = t\cdot e^{-5t} - e^{-5t} + e^{-6t}\ A$$

Unit-II Two Port Networks

A network consisting of two pairs of terminals is called two port network. The terminals are generally named as 1-1' and 2-2'. Generally a port designated 1-1', is connected to the driving energy source while the other port designated 2-2', is connected to the load. A port at which energy source is connected is called driving point of the network or Input port. A port at which load is connected is called as the Output port.



Two port Network parameters:-

A two port network consists of two ports (Pair of terminals) with two terminals on each port. consider such a two port network.

V_1, I_1 — Input port variables

V_2, I_2 — output port variables

Assumptions :-

① The voltages and current in the actual network inside box are not available for the measurements.

② The network inside the box is assumed to consist only the linear elements. Also the network may consist dependent sources but independent sources are not allowed.

③ If the network consists energy storing elements such as inductor and capacitor, then the initial conditions on them are assumed to be zero.

There are six different possible ways of selecting two independent variables of four variables. Thus six pairs of equations defining their own sets of parameters such as Z, Y, h, inverse h (g), transmission & inverse transmission parameters can be obtained.

① Z-Parameters (or) Open Circuit Impedance Parameters :-

These are also called as impedance parameters. They are expressed by Voltages in two ports interms of currents at two ports. Thus currents I_1 & I_2 are independent variables while V_1 & V_2 are dependent Variables.

$$V_1 = f_1(I_1, I_2) \quad \& \quad V_2 = f_2(I_1, I_2)$$

In equation form,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow V = Z \cdot I$$

Each individual parameters can be obtaind by assigning independent Variables to Zero.

① Let $I_2 = 0$; Port - 2 is open circuited

$$\therefore V_1 = Z_{11} I_1 \Rightarrow Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega$$

open circuit driving point input Impedance

$$\text{Also } V_2 = Z_{21} I_1 \Rightarrow Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega$$

open circuit forward transfer impedance.

② Let $I_1 = 0$; port - 1 is open circuited.

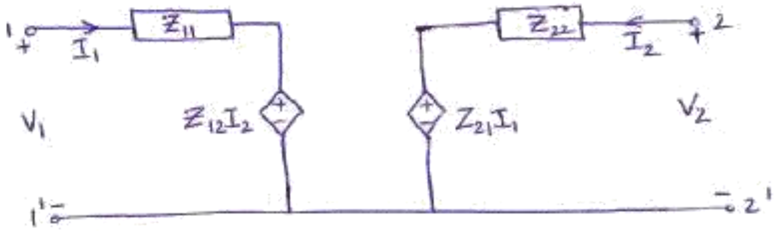
$$\therefore V_1 = Z_{12} I_2 \Rightarrow Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega$$

open circuit Reverse Transfer Impedance

$$\text{III}^{ly} \; V_2 = Z_{22} I_2 \Rightarrow Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega$$

open circuit driving point output Impedance.

Z parameters are also called as open circuit impedance parameters. This is because, at any of the instance one of the port is open circuited.



Equivalent network of a two port network interms of Z-Parameters

② Y-Parameters (or) Short Circuit Admittance parameters :-

These are also called admittance parameters. Expressed by currents at two ports interms of Voltages at two ports. Thus currents I_1 & I_2 are dependent variables and V_1 & V_2 are independent variables.

$$I_1 = f_1(V_1, V_2)$$
$$I_2 = f_2(V_1, V_2)$$

In Equation form

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y-parameters can be defined by considering independent variables to Zero.

① At $V_2 = 0$; ie port-2 is short circuited.

$$I_1 = Y_{11} V_1 \Rightarrow Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \mho$$ short circuit driving point input admittance

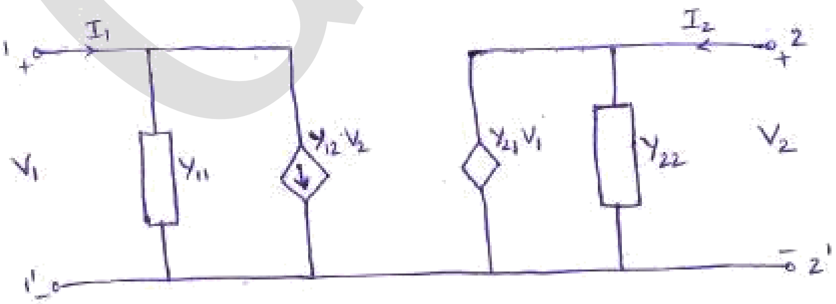
& $$I_2 = Y_{21} V_1 \Rightarrow Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \mho$$ Short circuit forward transfer admittance.

② At $V_1 = 0$; port-1 is short circuited.

$$\therefore I_1 = Y_{12} V_2 \Rightarrow Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \mho$$ Short circuit reverse transfer admittance

& $$I_2 = Y_{22} V_2 \Rightarrow Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \mho$$ Short circuit driving Point output admittance.

Y-Parameters are also called as short circuit admittance parameters. This is because, at any of the instance, one port is short circuited.



Equivalent network of a twoport network interms of Y-Parameters.

③ h - Parameters (or) Hybrid Parameters:-

These parameters are very useful in transistor modelling. The transistor parameters cannot be calculated using either short circuit admittance (or) open circuit impedance parameters measurements. They are expressed by Voltage at input port and current at output port interms of the current at input port and the voltage at output port. The current I_1 & Voltage V_2 are independent variables, while current I_2 and voltage V_1 are dependent variables.

$$V_1 = f_1(I_1, V_2)$$

$$I_2 = f_1(I_1, V_2)$$

In Equation form,

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Individual parameter can be defined as

① At $V_2 = 0$, Port 2 is short circuited.

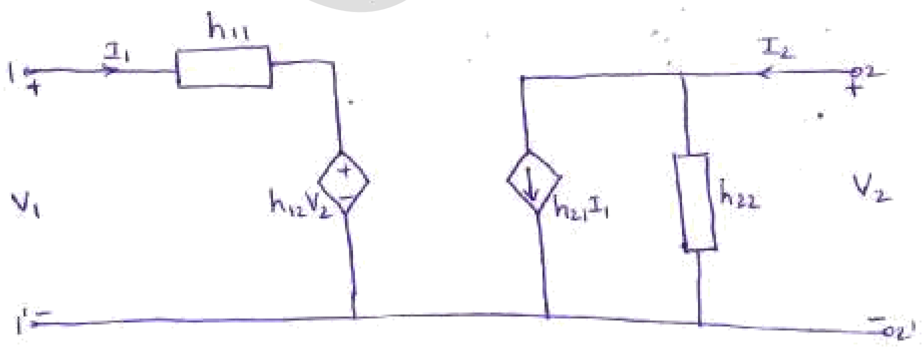
$V_1 = h_{11} I_1 \Rightarrow h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Ω Short circuit input impedance

$I_2 = h_{21} I_1 \Rightarrow h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ Short circuit forward current gain

② At $I_1 = 0$ Port-1 is open circuited

$V_1 = h_{12} V_2 \Rightarrow h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ open circuit reverse Voltage gain

$I_2 = h_{22} V_2 \Rightarrow h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ ℧ open circuit output admittance.



Equivalent network of a two port network in terms of h-Parameters.

④ ABCD Parameters (or) Transmission Parameters (or) Chain Parameters:-

These Parameters are known as transmission Parameters. These are generally used in the analysis of transmission of Power in which the input port is referred as the sending end while the output port is referred as receiving end. They are expressed by voltage V_1 & current I_1 at input port interms of the voltage V_2 and current I_2 at output port.

Thus voltage V_2 and current I_2 are independent variables while voltage V_1 and current I_1 are dependent variables.

$$V_1 = f_1(V_2, -I_2)$$

$$I_1 = f_2(V_2, -I_2)$$

Generally we consider the currents to be entering the port and are positive. For ABCD parameters, negative sign of I_2 indicates that current is leaving Port 2.

In equation Form,

$$V_1 = A V_2 + B(-I_2)$$

$$I_1 = C V_2 + D(-I_2)$$

In Matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The individual transmission parameters are defined as,

① $-I_2 = 0$; Port 2 is open circuited

$$V_1 = A V_2 \Rightarrow A = \left.\frac{V_1}{V_2}\right|_{-I_2=0}$$ open circuit reverse voltage gain

$$I_1 = C V_2 \Rightarrow C = \left.\frac{I_1}{V_2}\right|_{-I_2=0}$$ open circuit reverse transfer admittance

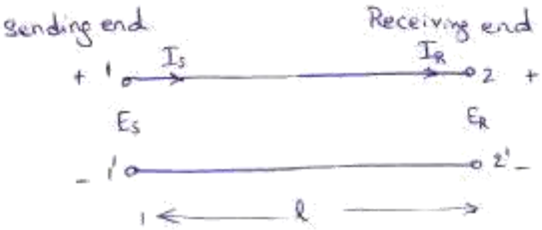
② $V_2 = 0$, Port 2 is short circuited,

$$V_1 = B(-I_2) \Rightarrow B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \Omega$$ Short circuit reverse transfer impedance

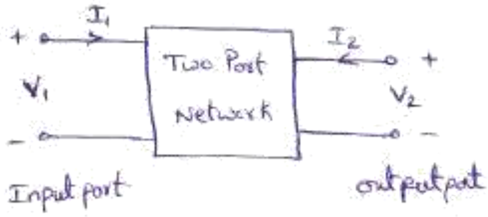
$$I_1 = D(-I_2) \Rightarrow D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$ Short circuit reverse current gain.

These ABCD Parameters are effectively used in analysis of Power transmission line. Input side is called as sending end and output side is called as receiving end. According to transmission line theory, sending end variables are expressed in terms of receiving end variables. ie., V_1 & I_1 are expressed in terms of V_2 and I_2

Due to this analogy ABCD parameters are also called as transmission parameters. They are also useful in analysis of two (or) more networks connected in cascade (or) chain. Hence these parameters are also called as chain parameters.



Transmission line



It is clear that the conventional current direction of I_R at output port in transmission line is away from the output port. But the direction of I_2 at output port in a two port network is positive ie, towards the network. Hence to have a analogy between transmission line and a general two port network, current I_2 is considered to be flowing away from the network, assumed to be $(-I_2)$.

Condition of Symmetry:-

If the impedance measured at one port is equal to the impedance measured at the other port with remaining port open circuited, the network is said to be symmetrical.

$V_1 = V_s$

$V_2 = V_s$

Conditions of symmetry of Z-Parameters:-

The basic equations of Z Parameters are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

For $V_1 = V_s$ & $I_2 = 0$

$$V_s = Z_{11} I_1 \Rightarrow \frac{V_s}{I_1} = Z_{11}$$

For $V_2 = V_s$ & $I_1 = 0$

$$\therefore V_s = Z_{22} I_2$$
$$\frac{V_s}{I_2} = Z_{22}$$

For condition of symmetry

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\Rightarrow \boxed{Z_{11} = Z_{22}}$$

Condition of Symmetry for Y-Parameters:-

The basic equation of Y-Parameters are,

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

For $V_1 = V_s$ & $I_2 = 0$

$$I_1 = Y_{11} V_s + Y_{12} V_2$$

|||ly $$0 = Y_{21} V_s + Y_{22} V_2$$

$$V_2 = \frac{-Y_{21}}{Y_{22}} V_s$$

Substitute V_2 in above eq of I_1,

$$I_1 = Y_{11} V_s + Y_{12}\left[\frac{-Y_{21}}{Y_{22}}\right] V_s$$
$$= V_s\left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{22}}\right]$$

$$\Rightarrow \frac{V_s}{I_1} = \left[\frac{Y_{22}}{Y_{11} Y_{22} - Y_{12} Y_{21}}\right]$$

For $V_2 = V_s$ & $I_1 = 0$

$$0 = Y_{11} V_1 + Y_{12} V_s$$
$$-Y_{11} V_1 = Y_{12} V_s$$
$$V_1 = \frac{-Y_{12}}{Y_{11}} V_s$$

Substitute V_1 in eq of I_2,

$$I_2 = Y_{21}\left(\frac{-Y_{12}}{Y_{11}}\right) V_s + Y_{22} V_s$$

$$\frac{V_s}{I_2} = \frac{Y_{11}}{Y_{11} Y_{22} - Y_{12} Y_{21}}$$

For condition of symmetry

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\Rightarrow \frac{Y_{22}}{\cancel{Y_{11} Y_{22} - Y_{12} Y_{21}}} = \frac{Y_{11}}{\cancel{Y_{11} Y_{22} - Y_{12} Y_{21}}}$$

$$\Rightarrow \boxed{Y_{11} = Y_{22}}$$

Condition of Symmetry for h-Parameters:-

The basic equation of h-Parameters,

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

For $V_1 = V_S$ & $I_2 = 0$

$$V_S = h_{11} I_1 + h_{12} V_2$$

III^{ly} $0 = h_{21} I_1 + h_{22} V_2$

$$-h_{22} V_2 = h_{21} I_1$$

$$V_2 = \frac{-h_{21}}{h_{22}} I_1$$

Substitute V_2 in V_S,

$$V_S = h_{11} I_1 + h_{12} \cdot \left(\frac{-h_{21}}{h_{22}}\right) I_1$$

$$\frac{V_S}{I_1} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}$$

For $V_2 = V_S$ & $I_1 = 0$

$$V_1 = h_{12} V_S$$

III^{ly} $I_2 = h_{22} V_S$

$$\frac{V_S}{I_2} = \frac{1}{h_{22}}$$

For condition of symmetry

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

$$\frac{h_{11} h_{22} - h_{12} h_{21}}{\cancel{h_{22}}} = \frac{1}{\cancel{h_{22}}}$$

$$\boxed{(h_{11} h_{22} - h_{12} h_{21}) = 1}$$

Condition of Symmetry for Transmission parameters:-

The basic equation of transmission parameters are,

$$V_1 = A V_2 + B (-I_2)$$

$$I_1 = C V_2 + D (-I_2)$$

For $V_1 = V_S$ & $I_2 = 0$

$$V_S = A V_2$$

III^{ly} $I_1 = C V_2$

$$V_2 = \frac{I_1}{C}$$

Substitute V_2 in V_S,

$$V_S = A \cdot \frac{I_1}{C}$$

$$\frac{V_S}{I_1} = \frac{A}{C}$$

For $V_2 = V_S$ & $I_1 = 0$

$$V_1 = A V_S + B(-I_2)$$

III^{ly} $0 = C V_S + D(-I_2)$

$$C V_S = D I_2$$

$$\frac{V_S}{I_2} = \frac{D}{C}$$

For condition of symmetry,

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

$$\frac{A}{\cancel{C}} = \frac{D}{\cancel{C}}$$

$$\boxed{A = D}$$

Condition of Reciprocity:-

If the ratio of voltage at one port to the current at other port is same to the ratio of, if voltage and current positions are interchanged, then network is said to be reciprocal.

For Z Parameters:-

$V_1 = V_s$; $V_2 = 0$; $I_2' = -I_2$

$V_2 = V_s$; $V_1 = 0$; $I_1' = -I_1$

The basic equations,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

For $V_1 = V_s$, $V_2 = 0$ & $I_2' = -I_2$

$$V_s = Z_{11} I_1 + Z_{12}(-I_2')$$

III^{ly} $0 = Z_{21} I_1 + Z_{22}(-I_2')$

$$Z_{21} I_1 = Z_{22} I_2'$$

$$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$$

Substitute I_1 in Eq of V_s,

$$V_s = \frac{Z_{11} \cdot Z_{22}}{Z_{21}} I_2' + Z_{12}(-I_2')$$

$$\frac{V_s}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

For $V_2 = V_s$; $V_1 = 0$; $I_1' = -I_1$

$$0 = Z_{11}(-I_1') + Z_{12} I_2$$

$$Z_{11} I_1' = Z_{12} I_2$$

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1'$$

Substitute I_2 in Eq of V_2

$$V_s = -Z_{21} I_1' + Z_{22}\left[\frac{Z_{11}}{Z_{12}}\right] I_1'$$

$$\frac{V_s}{I_1'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{12}}$$

For condition of Reciprocity,

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

$$\Rightarrow \boxed{Z_{12} = Z_{21}}$$

For Y Parameters:-

The basic equations,

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

For $V_1 = V_s$; $V_2 = 0$; $I_2' = -I_2$

$$I_1 = Y_{11} V_s$$

III^{ly} $-I_2' = Y_{21} V_s$

$$\frac{V_s}{I_2'} = -\frac{1}{Y_{21}}$$

For $V_2 = V_s$; $V_1 = 0$; $I_1' = -I_1$

$$-I_1' = Y_{12} V_s$$

III^{ly} $I_2 = Y_{22} V_s$

$$\frac{V_s}{I_1'} = \frac{-1}{Y_{12}}$$

For condition of Reciprocity

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

$$-\frac{1}{Y_{12}} = -\frac{1}{Y_{21}}$$

$$\boxed{Y_{12} = Y_{21}}$$

Conversion of one parameters to other parameters :-

① **Z Parameters :**

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

ⓐ In terms of Y Parameters :-

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using Cramer's rule for V_1 & V_2,

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22} I_1 - Y_{12} I_2}{Y_{11} Y_{22} - Y_{12} Y_{21}} = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2$$

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{-Y_{21} I_1 + Y_{11} I_2}{Y_{11} Y_{22} - Y_{12} Y_{21}} = \frac{-Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2$$

$$\Rightarrow V_1 = \left[\frac{Y_{22}}{\Delta Y}\right] I_1 + \left[\frac{-Y_{12}}{\Delta Y}\right] I_2 \quad \& \quad V_2 = \left[\frac{-Y_{21}}{\Delta Y}\right] I_1 + \left[\frac{Y_{11}}{\Delta Y}\right] I_2$$

comparing with Z Parameters

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

ⓑ In terms of h-parameters :-

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow h_{22} V_2 = -h_{21} I_1 + I_2$$

$$V_2 = \left[\frac{-h_{21}}{h_{22}}\right] I_1 + \left[\frac{1}{h_{22}}\right] I_2$$

Illly $$V_1 = h_{11} I_1 + h_{12} \left[\left(\frac{-h_{21}}{h_{22}}\right) I_1 + \left(\frac{1}{h_{22}}\right) I_2\right]$$

$$= \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right] I_1 + \left[\frac{h_{12}}{h_{22}}\right] I_2$$

Therefore,

$$[Z] = \begin{bmatrix} \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

$$V_1 = A V_2 + B(-I_2)$$
$$I_1 = C V_2 + D(-I_2)$$

$$\Rightarrow C V_2 = I_1 + D I_2$$

$$V_2 = \left[\frac{1}{C}\right] I_1 + \left[\frac{D}{C}\right] I_2$$

$$\Rightarrow V_1 = A\left[\left(\frac{1}{C}\right) I_1 + \left(\frac{D}{C}\right) I_2\right] + B(-I_2) = \left(\frac{A}{C}\right) I_1 + \left(\frac{AD}{C} - B\right) I_2$$

$$\therefore [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{AD}{C} - B \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

2) Y Parameters :-

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

a) Interms of Z-parameters :-

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Using Cramer's Rule for I_1 & I_2,

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{22}}{\Delta Z} V_1 + \frac{(-Z_{12})}{\Delta Z} V_2$$

|||ly
$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{(-Z_{21})}{\Delta Z} V_1 + \frac{Z_{11}}{\Delta Z} V_2$$

$$\Rightarrow [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Y} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

(b) interms of h-Parameters:-

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\Rightarrow h_{11} I_1 = V_1 - h_{12} V_2$$

$$I_1 = \left(\frac{1}{h_{11}}\right) V_1 + \left(\frac{-h_{12}}{h_{11}}\right) V_2$$

$$\Rightarrow I_2 = h_{21}\left[\left(\frac{1}{h_{11}}\right)V_1 + \left(\frac{-h_{12}}{h_{11}}\right)V_2\right] + h_{22} V_2 = \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right) V_2$$

$$\Rightarrow [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{h_{11}h_{22} - h_{21}h_{12}}{h_{11}} \end{bmatrix}$$

(c) In terms of ABCD parameters :-

$$V_1 = A V_2 + B(-I_2)$$

$$I_1 = C V_2 + D(-I_2)$$

$$\Rightarrow -B I_2 = V_1 - A V_2 \Rightarrow I_2 = \left(\frac{-1}{B}\right) V_1 + \left(\frac{A}{B}\right) V_2$$

Substitute I_2 value in Eq of I_1

$$I_1 = C V_2 + D\left[\frac{1}{B} V_1 - \frac{A}{B} V_2\right]$$

$$= \left(\frac{BC - AD}{B}\right) V_2 + \left(\frac{D}{B}\right) V_1 = \left(\frac{D}{B}\right) V_1 + \left(\frac{BC-AD}{B}\right) V_2$$

$$\Rightarrow Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \left(\frac{D}{B}\right) & \left(\frac{BC-AD}{B}\right) \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$

3) h-Parameters :-

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

ⓐ Interms of Z-Parameters:-

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{22} I_2 = -Z_{21} I_1 + V_2$$

$$I_2 = \left[\frac{-Z_{21}}{Z_{22}}\right] I_1 + \left[\frac{1}{Z_{22}}\right] V_2$$

Substitute I_2 in V_1 eq,

$$V_1 = Z_{11} I_1 + Z_{12}\left[\frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right] = \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}\right] I_1 + \left[\frac{Z_{12}}{Z_{22}}\right] V_2$$

$$\Rightarrow [h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

ⓑ Interms of Y-Parameters:-

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\Rightarrow y_{11} V_1 = I_1 - y_{12} V_2$$
$$V_1 = \left(\frac{1}{y_{11}}\right) I_1 + \left(\frac{-y_{12}}{y_{11}}\right) V_2$$

Substitute V_1 in Eq of I_2,

$$I_2 = y_{21}\left[\frac{1}{y_{11}} I_1 + \left(\frac{-y_{12}}{y_{11}}\right) V_2\right] + y_{22} V_2 = \left(\frac{y_{21}}{y_{11}}\right) I_1 + \left(\frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}}\right) V_2$$

Hence

$$h = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}} \end{bmatrix}$$

© **Interms of ABCD Parameters:-**

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

$$DI_2 = -I_1 + CV_2$$

$$I_2 = \left(\frac{-1}{D}\right)I_1 + \left(\frac{C}{D}\right)V_2$$

Substitute I_2 in Eq of V_1,

$$V_1 = AV_2 - B\left[-\frac{1}{D}I_1 + \frac{C}{D}V_2\right]$$

$$= \left(\frac{AD-BC}{D}\right)V_2 + \frac{B}{D}I_1 = \left(\frac{B}{D}\right)I_1 + \left(\frac{AD-BC}{D}\right)V_2$$

$$\Rightarrow [h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$$

4) ABCD Parameters:-

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

ⓐ **Interms of Z Parameters:-**

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{21}I_1 = V_2 - Z_{22}I_2 \Rightarrow I_1 = \left(\frac{1}{Z_{21}}\right)V_2 + \left(\frac{-Z_{22}}{Z_{21}}\right)I_2$$

Substitute I_1 in Eq of V_1,

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 + \frac{-Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2$$

$$= \frac{Z_{11}}{Z_{21}}V_2 + \left(\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right)(-I_2)$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

b) In terms of Y-Parameters:-

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{21} V_1 = -Y_{22} V_2 + I_2 \Rightarrow V_1 = \left(\frac{-Y_{22}}{Y_{21}}\right) V_2 + \left(\frac{-1}{Y_{21}}\right)(-I_2)$$

Substitute the value of V_1 in Eq of I_1,

$$I_1 = Y_{11}\left[\frac{-Y_{22}}{Y_{21}} V_2 + \left(\frac{-1}{Y_{21}}\right)(-I_2)\right] + Y_{12} V_2$$

$$= \left(\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right) V_2 + \left(\frac{-Y_{11}}{Y_{22}}\right)(-I_2)$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

c) In terms of h-Parameters:-

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{21} I_1 = -h_{22} V_2 + I_2 \Rightarrow I_1 = \frac{-h_{22}}{h_{21}} V_2 + \left(\frac{-1}{h_{21}}\right)(-I_2)$$

Substitute value of I_1 in Eq of V_1,

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}} V_2 + \left(\frac{-1}{h_{21}}\right)(-I_2)\right] + h_{12} V_2$$

$$\Rightarrow V_1 = \left(\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right) V_2 + \left(\frac{-h_{11}}{h_{21}}\right)(-I_2)$$

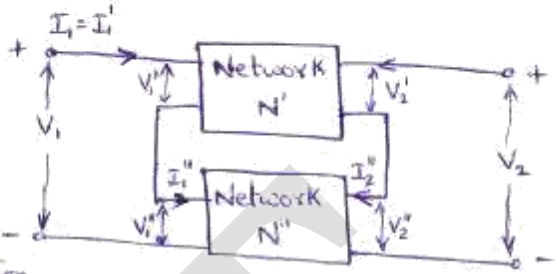
$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$$

Interconnection of Two ports:-

1. Series connection of two ports
2. Parallel connection of two ports
3. Cascade connection of two ports

1) Series Connection of two ports:-

Consider that two port Networks N' & N'' are connected in series. When two ports are connected in series, we can add their z parameters to get overall Z parameters of the connection.



Let z parameters of N' be $Z_{11}', Z_{12}', Z_{21}', Z_{22}'$ &

N'' be $Z_{11}'', Z_{12}'', Z_{21}'', Z_{22}''$

Let overall Z Parameters of series connection be $Z_{11}, Z_{12}, Z_{21}, Z_{22}$

For series connection, $V_1 = V_1' + V_1''$ — ①; $V_2 = V_2' + V_2''$ — ② &

$I_1 = I_1' = I_1''$ — ③; $I_2 = I_2' = I_2''$ — ④

For Network N',

$$V_1' = Z_{11}' I_1' + Z_{12}' I_2'$$
$$V_2' = Z_{21}' I_1' + Z_{22}' I_2'$$

For Network N'',

$$V_1'' = Z_{11}'' I_1'' + Z_{12}'' I_2''$$
$$V_2'' = Z_{21}'' I_1'' + Z_{22}'' I_2''$$

From the equations ① ② & ③ ④

$$V_1 = (Z_{11}' + Z_{11}'') I_1 + (Z_{12}' + Z_{12}'') I_2$$
$$V_2 = (Z_{21}' + Z_{21}'') I_1 + (Z_{22}' + Z_{22}'') I_2$$

In Matrix Form,

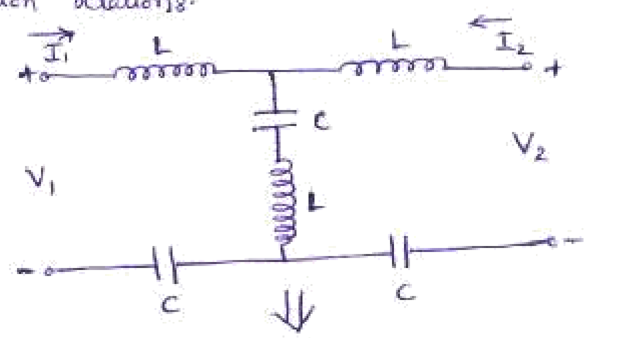
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (Z_{11}' + Z_{11}'') & (Z_{12}' + Z_{12}'') \\ (Z_{21}' + Z_{21}'') & (Z_{22}' + Z_{22}'') \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

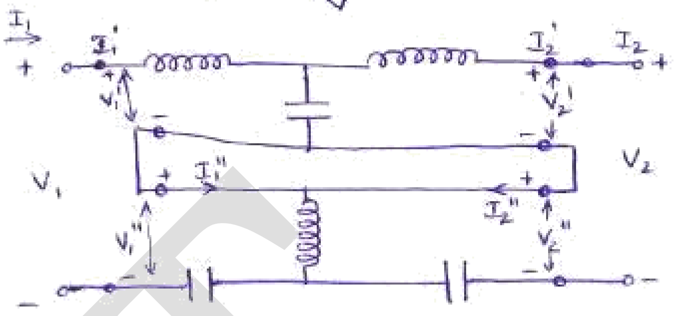
overall Z-Parameters,

$$[Z] = \begin{bmatrix} (Z_{11}' + Z_{11}'') & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix}$$

Problem:-

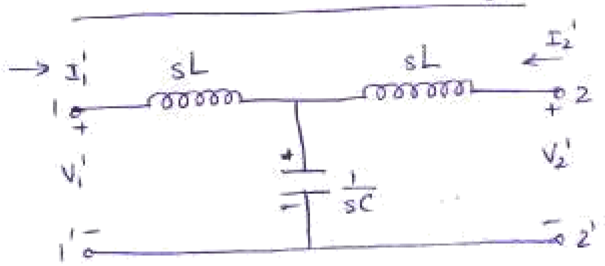
① Find Z Parameters by using interconnection relations.





Sol:-

Ⓐ To determine Z Parameters of N' :-



Network in S-domain,

① Let $I_2' = 0$, ie., Port-2 open circuited,

Apply KVL to ~~input~~ input side,

$$V_1' = I_1'\left[sL + \frac{1}{sC}\right] \Rightarrow \frac{V_1'}{I_1'} = \left(sL + \frac{1}{sC}\right)\Omega$$

by definition, $Z_{11}' = \frac{V_1'}{I_1'} = \left(sL + \frac{1}{sC}\right)\ \Omega$

$|||^{ly}$ Voltage V_2' at output side in terms of I_1' is

$$V_2' = \left(\frac{1}{sC}\right)I_1' \Rightarrow \frac{V_2'}{I_1'} = \frac{1}{sC}$$

by definition, $Z_{21}' = \frac{V_2'}{I_1'} = \frac{1}{sC}\ \Omega$

② to determine Z_{12}' & Z_{22}',

Let $I_1' = 0$ ie., Port 1 is open circuited,

Apply KVL to output side,

$$V_2' = I_2'\left(sL + \frac{1}{sC}\right) \Rightarrow \frac{V_2'}{I_2'} = \left(sL + \frac{1}{sC}\right)$$

by definition $Z_{22}' = \left(sL + \frac{1}{sC}\right)\ \Omega$

$|||^{ly}$ Voltage V_1' in terms of I_2' is

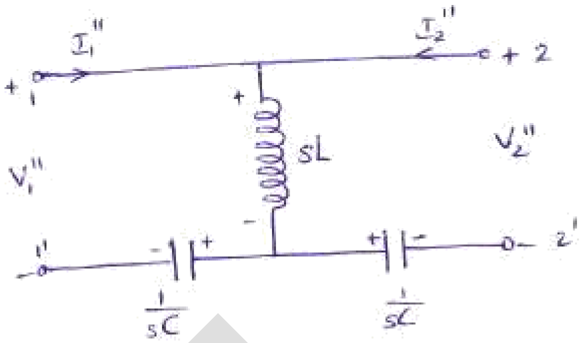
$$V_1' = \left(\frac{1}{sC}\right)I_2' \Rightarrow \frac{V_1'}{I_2'} = \left(\frac{1}{sC}\right)$$

by definition $Z_{12}' = \left(\frac{1}{sC}\right)\ \Omega$

∴ For the network N',

$$[Z'] = \begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix} = \begin{bmatrix} sL+\frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & sL+\frac{1}{sC} \end{bmatrix}$$

(B) To determine z parameters of N'' :-



Network N'' in s-domain.

(1) To determine Z_{11}'' & Z_{21}''

let $I_2''=0$; ie, Port 2 open circuited.

Apply KVL,

$$V_1'' = \left(sL+\frac{1}{sC}\right) I_1''$$

$$\frac{V_1''}{I_1''} = \left(sL+\frac{1}{sC}\right)$$

by definition, $Z_{11}'' = \frac{V_1''}{I_1''} = \left(sL+\frac{1}{sC}\right)\ \Omega$

IIIly Voltage V_2'' in terms of I_1'' is

$$V_2'' = (sL)\ I_1'' \Rightarrow \frac{V_2''}{I_1''} = sL$$

By definition, $Z_{21}'' = \frac{V_2''}{I_1''} = (sL)\ \Omega$

(2) To determine Z_{12}'' & Z_{22}'', let $I_1''=0$; ie, Port 1 open circuited,

Apply KVL,

$$V_2'' = \left(sL+\frac{1}{sC}\right) I_2'' \Rightarrow \frac{V_2''}{I_2''} = \left(sL+\frac{1}{sC}\right)$$

by definition $Z_{22}'' = \frac{V_2''}{I_2''} = \left(sL+\frac{1}{sC}\right)\ \Omega$

IIIly V_1'' in terms of I_2'' is

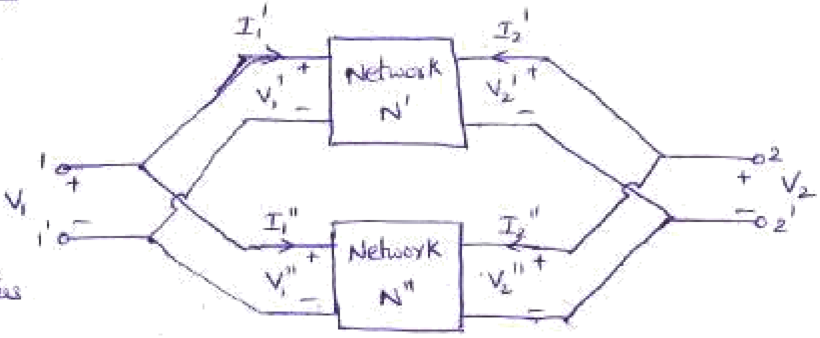
$$V_1'' = (sL)\ I_2'' \Rightarrow \frac{V_1''}{I_2''} = (sL)$$

by definition $Z_{12}'' = \frac{V_1''}{I_2''} = (sL)\ \Omega$

$$[Z''] = \begin{bmatrix} sL+\frac{1}{sC} & sL \\ sL & sL+\frac{1}{sC} \end{bmatrix}$$

$$[Z] = [Z']+[Z''] = \begin{bmatrix} 2\left(sL+\frac{1}{sC}\right) & sL+\frac{1}{sC} \\ sL+\frac{1}{sC} & 2\left(sL+\frac{1}{sC}\right) \end{bmatrix} = \left(sL+\frac{1}{sC}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

2) Parallel connection of two Ports:-



Consider two networks N' & N'' are connected in parallel. when two ports are connected in parallel, we can add their Y-Parameters to get overall Y-Parameters of the Parallel connection.

Let

Network, N' $\longrightarrow$ Y_{11}', Y_{12}', Y_{21}' & Y_{22}'

N'' $\longrightarrow$ Y_{11}'', Y_{12}'', Y_{21}'' & Y_{22}''

& overall Y Parameters of Parallel connection, Y_{11}, Y_{12}, Y_{21}, Y_{22}

For Parallel connection,

$$I_1 = I_1' + I_1'' \quad (1) \quad \& \quad I_2 = I_2' + I_2'' \quad (2)$$

$$\& \quad V_1 = V_1' = V_1'' \quad (3), \quad V_2 = V_2' = V_2'' \quad (4)$$

For Network N',

$$I_1' = Y_{11}'V_1' + Y_{12}'V_2'$$

$$I_2' = Y_{21}'V_1' + Y_{22}'V_2'$$

For Network N'',

$$I_1'' = Y_{11}''V_1'' + Y_{12}''V_2''$$

$$I_2'' = Y_{21}''V_1'' + Y_{22}''V_2''$$

From the equations (1) (2) & (3) (4)

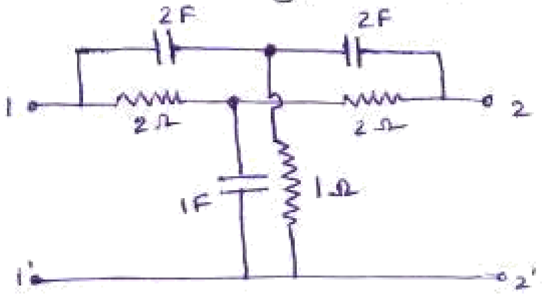
$$I_1 = (Y_{11}' + Y_{11}'')V_1 + (Y_{12}' + Y_{12}'')V_2$$

$$I_2 = (Y_{21}' + Y_{21}'')V_1 + (Y_{22}' + Y_{22}'')V_2$$

In matrix Form,

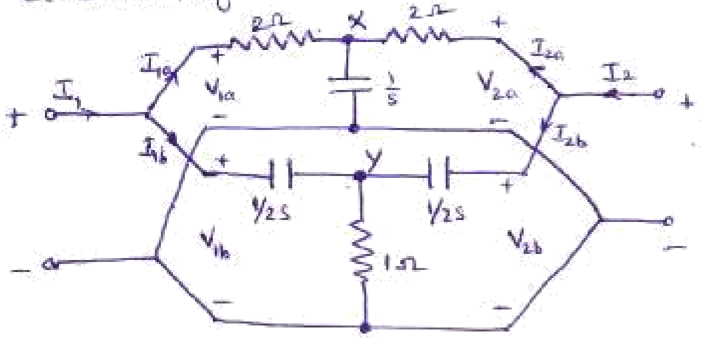
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Problem:- The network of the shown in the figure, is of the type used for the "notch filter". For the element values given determine the y parameters.



Sol:-

Given network may be redrawn as parallel combination of two networks



Transformed Network

$\Rightarrow$ $V_1 = V_{1a} = V_{1b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$

$I_2 = I_{2a} + I_{2b}$

Equation of Y Parameters:-

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Apply KCL at node x', (2Ω, 2Ω, 1F)

$$I_{1a} + I_{2a} = \frac{V_x}{\left(\frac{1}{s}\right)} = s.V_x$$

$$\frac{V_{1a}-V_x}{2} + \frac{V_{2a}-V_x}{2} = s V_x$$

$$\frac{V_{1a}+V_{2a}}{2} = (s+1) V_x$$

$$\Rightarrow V_x = \frac{V_{1a}+V_{2a}}{2(s+1)}$$

Substituting this value in Eq of I_{1a}

$$I_{1a} = \frac{V_{1a} - V_x}{2} = \frac{V_{1a}}{2} - \frac{1}{2}\left[\frac{V_{1a}+V_{2a}}{2(s+1)}\right]$$

$$I_{1a} = \left[\frac{1}{2} - \frac{1}{4(s+1)}\right] V_{1a} - \left[\frac{1}{4(s+1)}\right] V_{2a}$$

III^{ly} Substitute V_x in I_{2a} Equation,

$$I_{2a} = \frac{V_{2a}-V_x}{2} = \frac{V_{2a}}{2} - \frac{1}{2}\left[\frac{V_{1a}+V_{2a}}{2(s+1)}\right]$$

$$\Rightarrow I_{2a} = \left[\frac{-1}{4(s+1)}\right] V_{1a} + \left[\frac{1}{2} - \frac{1}{4(s+1)}\right] V_{2a}$$

$\therefore$ Y Parameters of the Network,

$$[Y_{1a}] = \begin{bmatrix} \frac{1}{2} - \frac{1}{4(s+1)} & \frac{-1}{4(s+1)} \\ \frac{-1}{4(s+1)} & \frac{1}{2} - \frac{1}{4(s+1)} \end{bmatrix}$$

Now by applying KCL for node Y, (2F, 2F & 1Ω)

$$I_{1b} + I_{2b} = \frac{V_y}{1}$$

$$\frac{V_{1b} - V_y}{\left(\frac{1}{2s}\right)} + \frac{V_{2b} - V_y}{\left(\frac{1}{2s}\right)} = V_y$$

$$2s(V_{1b} - V_y) + 2s(V_{2b} - V_y) = V_y$$

$$2s\left[V_{1b} + V_{2b}\right] = (1+4s)V_y$$

$$V_y = \frac{2s(V_{1b} + V_{2b})}{1+4s}$$

Substituting V_y in Eq of I_{1b}

$$I_{1b} = \frac{V_{1b} - V_y}{(1/2s)} = 2s(V_{1b} - V_y) = 2s\left[V_{1b} - \frac{2s(V_{1b}+V_{2b})}{1+4s}\right]$$

$$\Rightarrow I_{1b} = \left[2s - \frac{4s^2}{1+4s}\right]V_{1b} - \left[\frac{4s^2}{1+4s}\right]V_{2b}$$

||ly substituting V_y in Eq of I_{2b}

$$I_{2b} = \frac{V_{2b} - V_y}{(1/2s)} = 2s(V_{2b} - V_y) = 2s\left[V_{2b} - \frac{2s(V_{1b}+V_{2b})}{1+4s}\right]$$

$$\Rightarrow I_{2b} = \left[\frac{-4s^2}{1+4s}\right]V_{1b} + \left[2s - \frac{4s^2}{1+4s}\right]V_{2b}$$

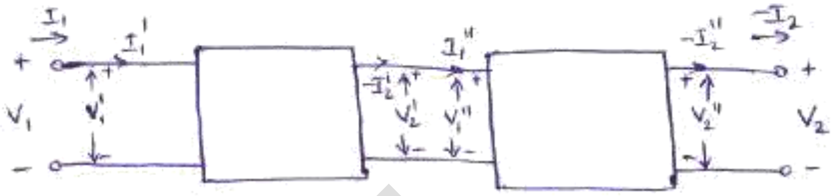
$$Y_{1b} = \begin{bmatrix} 2s - \frac{4s^2}{1+4s} & \frac{-4s^2}{1+4s} \\ -\frac{4s^2}{1+4s} & 2s - \frac{4s^2}{1+4s} \end{bmatrix}$$

∴ Total Y Parameters is given by

$$Y =$$

3) Cascade connection of Two Ports:-

The cascade connection is also called Tandem Connection. Consider two Networks N' & N'' are connected in cascade as shown. When two ports are connected in cascade, we can multiply their individual transmission parameters to get the overall transmission parameters of the cascade connection.



Let Transmission parameters of Network N' be A', B', C', D' & N'' be A'', B'', C'', D''

For cascade connection, we have

$$V_1 = V_1', \quad V_2' = V_1''; \quad V_2 = V_2''$$

$$I_1' = I_1; \quad -I_2' = I_1''; \quad -I_2 = -I_2''$$

For Network N',

$$V_1' = A'V_2' + B'(-I_2')$$

$$I_1' = C'V_2' + D'(-I_2')$$

For Network N'',

$$V_1'' = A''V_2'' + B''(-I_2'')$$

$$I_1'' = C''V_2'' + D''(-I_2'')$$

Overall Transmission parameters of the cascade connection are

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix}$$

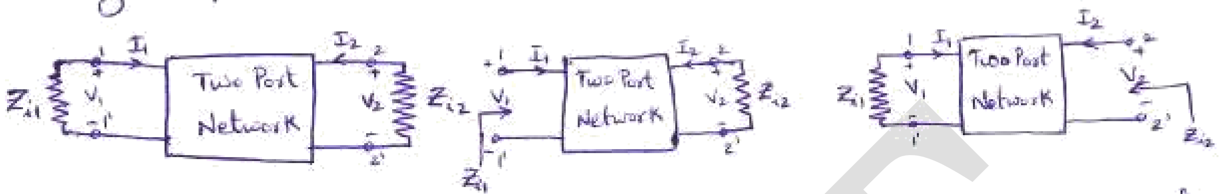
$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Image Parameters:-

In a two port network, if impedance measured at Port-1 is Z_{i1} with port 2 terminated in Z_{i2} & if impedance measured at Port-2 is Z_{i2} with Port 1 terminated in Z_{i1}, then two impedances are called as image impedances.



Assume that Z_{i1} & Z_{i2} are image impedances at Port ① & Port ② respectively.

If Port-2 is terminated into image impedance at that port ie, Z_{i2}, then impedance measured at that port-1 is equal to the image impedance at that Port, Z_{i1}.

Ill'y If Port-1 is terminated into image impedance at that port ie, Z_{i1}, then impedance measured at Port-2 is equal to the image impedance at that Port, Z_{i2}.

$\Rightarrow$ Z_{i1} = Driving point impedance at Port-1 = $\frac{V_1}{I_1}$

Z_{i2} = Driving point impedance at port-2 = $\frac{V_2}{I_2}$

Image parameters in terms of open & Short circuit impedances:-

$$Z_{1oc} = \frac{A}{C} \quad ; \quad Z_{2oc} = \frac{D}{C} \quad ; \quad Z_{1sc} = \frac{B}{D} \quad ; \quad Z_{2sc} = \frac{B}{A}$$

By definition, image impedance at any port is the geometric mean of o.c & s.c impedances at that port.

$$Z_{i1} = \sqrt{Z_{1oc} \cdot Z_{1sc}} \quad ; \quad Z_{i2} = \sqrt{Z_{2oc} \cdot Z_{2sc}}$$

Interms of ABCD:- $Z_{i1} = \sqrt{\frac{A}{C} \cdot \frac{B}{D}} \quad ; \quad Z_{i2} = \sqrt{\frac{D}{C} \cdot \frac{B}{A}}$

Image transfer constant, $\theta = \tan^{-1}h\sqrt{\frac{Z_{1sc}}{Z_{1oc}}} = \tan^{-1}h\sqrt{\frac{Z_{2sc}}{Z_{2oc}}} = \tan^{-1}h\sqrt{\frac{BC}{AD}}$

In general, θ is a complex quantity consisting real part – Image attenuation Constant
Imaginary part – Image phase constant

Thus Z_{i1}, Z_{i2} & θ are called as image parameters of two port Network.

A network which freely passes desired band of frequencies, while almost suppresses other band of frequencies is called filter. In filters, attenuation changes suddenly as the frequency is varies. Thus filters have the ability to discriminated between signals which differ by frequency.

Main classification of filters:-

1. Active filters : Active elements – Transistors, op-amps along with RLC, Voltage, current & Power gain Possible. They require additional power for their operation.
2. Passive filters:- only RLC – ie, Passive elements ; V, I, P gains not possible. They donot require additional power for operation. But as Inductors are bulky, They are costly.

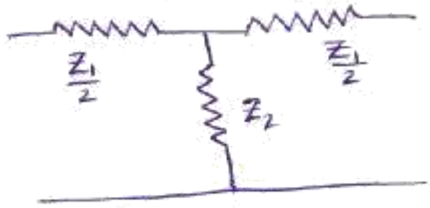
Basic filter Networks:-

The range of frequencies over which attenuation is zero is called passband.

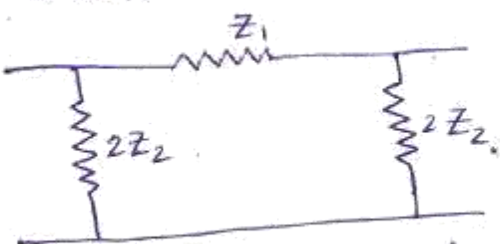
" " " " " " is infinite → Stop band (or) Attenuation band

The frequencies which seperate pass band from attenuation band are called as cut-off frequencies, denoted by f_c.

Symmetrical T & π sections:- (unbalanced)



Symmetrical unbalanced T section



Symmetrical unbalanced π section.

The importantproperties of symmetrical networks are

① characteristic Impedance (Z_0) &

② Propagation constant (γ)

For T - Network:-

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad \& \quad e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_{0T}}{Z_2}$$

For π - Network:-

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}} = \frac{Z_1 Z_2}{Z_{0T}} \; ; \quad e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_{0\pi}}$$

Types of filters:-

① If filter passes all frequencies upto cutoff frequency & attenuates all frequencies above it, then is called low pass filter

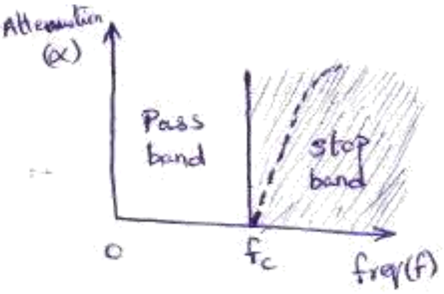
② If filter attenuates all frequences upto cutoff frequency & passes all frequencies above it, then is called High pass filter

In general types of filters, they have one pass band, one stop band & a single cutoff frequency. But we can design with two cutoff frequencies to get two more filter sections.

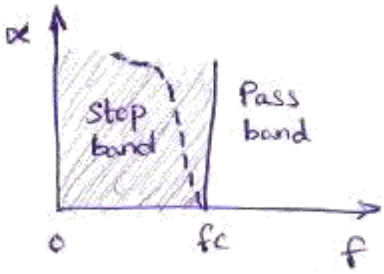
③ If filter passes all the frequencies between the two cutoff frequencies & attenuates all other frequencies, then is called Band pass filter.

④ If filter attenuates all the frequencies between the two cutoff frequencies & Passes all other frequencies, then it is called as Band stop or Elimination Filter.

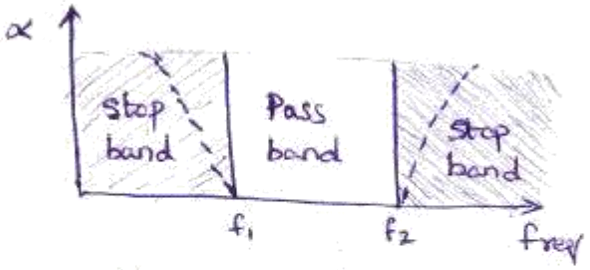
An ideal filter ~~also~~ would have zero attenuation in pass band & infinite attenuation in stop band. But practically, stop band attenuation gradually changes. It cannot change zero to infinite or infinite to zero instantaneously.



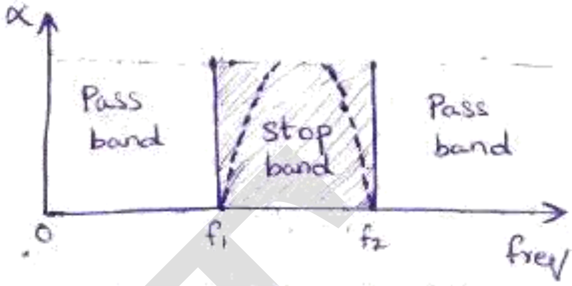
(a) Low pass filter



(b) High pass filter



(c) Band pass filter



(d) Band Elimination filter

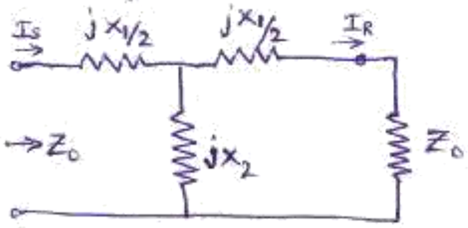
Characteristic Impedance in Pass band & Stop band:-

In order to determine the characteristic impedance of Pass & Stop band filters, one need to always consider a filter of reactive elements (Pure). The value of Z_0 varies with the reactances Z_1 & Z_2 offered by purely reactive elements that are used in series & shunt arms of a filter.

Theorem:

"Over the range of frequencies for which the characteristic impedance Z_0 of a filter is purely resistive (real), the attenuation constant α is zero. Over the range of frequencies for which Z_0 is purely reactive (imaginary), the α value is greater than zero."

Consider a 'T' Network with all the elements reactive, represented by jX where X is real, but may be +ve (or) -ve.



$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad ; \quad e^{\gamma} = \frac{I_S}{I_R} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

$$\text{and } \alpha = 20 \log_{10} \left| \frac{I_S}{I_R} \right| \text{ db}$$

Here, $Z_1 = jX_1$ & $Z_2 = jX_2$

$$\therefore Z_0 = \sqrt{\frac{(jX_1)^2}{4} + (jX_1)(jX_2)} = \sqrt{\frac{-X_1^2}{4} - X_1 X_2} = \sqrt{-\left(\frac{X_1^2}{4} + X_1 X_2\right)} = j\sqrt{\frac{X_1^2}{4} + X_1 X_2}$$

$$e^{\gamma} = \frac{I_S}{I_R} = 1 + \frac{X_1}{2X_2} + \frac{Z_0}{jX_2} = \left(1 + \frac{X_1}{2X_2}\right) - j\left(\frac{Z_0}{X_2}\right)$$

depending on the signs of X_1 & X_2, we get

Case ① $\frac{X_1^2}{4} + X_1X_2$ is negative $\Rightarrow -A$

Case ② $\frac{X_1^2}{4} + X_1X_2$ is positive $\Rightarrow +B$

where A & B are real & positive

Case ①:-

If $\frac{X_1^2}{4} + X_1X_2 = -A$ then $Z_0 = j\sqrt{\frac{X_1^2}{4} + X_1X_2} = j\sqrt{-A} = +\sqrt{A}$

Here Z_0 is real & purely resistive,

$$e^{\gamma} = \frac{I_S}{I_R} = \left[1 + \frac{X_1}{2X_2}\right] - j\left[\frac{\sqrt{A}}{X_2}\right]$$

$$\left|\frac{I_S}{I_R}\right| = \sqrt{\left(1+\frac{X_1}{2X_2}\right)^2 + \left(\frac{\sqrt{A}}{X_2}\right)^2} = \sqrt{\left(1+\frac{X_1}{2X_2}\right)^2 + \frac{A}{X_2^2}}$$

$$\Rightarrow \sqrt{\left(1+\frac{X_1^2}{4X_2^2}\right) + \frac{X_1}{X_2} + \frac{-\left(\frac{X_1^2}{4}+X_1X_2\right)}{X_2^2}} = \sqrt{1 + \frac{X_1}{X_2} + \frac{X_1^2}{4X_2^2} - \frac{X_1^2}{4X_2^2} - \frac{X_1}{X_2}} = 1$$

$$\alpha = 20\log_{10}\left|\frac{I_S}{I_R}\right| = 20\log_{10}(1) = 0 \text{ if } Z_0 \text{ is real}$$

Case ②:- If $\frac{X_1^2}{4} + X_1X_2 = +B$ then $Z_0 = j\sqrt{\frac{X_1^2}{4} + Z_1Z_2} = j\sqrt{B}$

Here Z_0 is imaginary (Purely reactive)

$$e^{\gamma} = \frac{I_S}{I_R} = \left[1 + \frac{X_1}{2X_2}\right] - j\left[\frac{j\sqrt{B}}{X_2}\right] = \left(1 + \frac{X_1}{2X_2}\right) + \frac{\sqrt{B}}{X_2}$$

$$\Rightarrow \left|\frac{I_S}{I_R}\right| = 1 + \frac{X_1}{2X_2} + \frac{\sqrt{\frac{X_1^2}{4}+X_1X_2}}{X_2}$$ The ratio is real & greater than 1.

Thus α is not equal to zero, but greater than zero & Z_0 is imaginary.

Hence in a filter, over a range of frequencies, Z_0 may be either real or imaginary.

→ When Z_0 is real, the filter and its terminating impedance will absorb power from any generator connected to it. Since filter composed of ~~reactive~~ reactive elements, it cannot by itself absorb power. Instead power delivered by generator passes to the load. Thus there is no attenuation i.e., $\alpha = 0$; This indicates <u>Pass band</u>

→ When Z_0 is imaginary or Purely reactive, the filter & its termination cannot absorb any power. Thus no power is passed to the load. Thus attenuation is very high, ideally it is infinite. This indicates <u>stop Band</u>

$\sinh\frac{\alpha}{2} \cdot \cos\frac{\beta}{2} = 0$ & $\cosh\frac{\alpha}{2} \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

Case ①

$\sinh\frac{\alpha}{2} = 0$

$\alpha = 0;\ \beta \neq 0$

$\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

Case ②

$\cos\frac{\beta}{2} = 0$

$\beta = (2n-1)\pi$ & $\sin\frac{\beta}{2} = \pm 1$

Also As $\alpha \neq 0$

$\cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

Case① gives pass band

$\because \alpha = 0$ & limited by $\sin\frac{\beta}{2} = 1$

$-1 < \frac{Z_1}{4Z_2} < 0$

$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$ — ①

Case② gives stop band (i.e) when $\alpha \neq 0$

The phase angle is π &

$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \longrightarrow$ ②

These Equation are important, as we can calculate attenuation α in the stopband and phase shift β in the passband where $\alpha = 0$.

Constant K Sections :-

for a T or π section in which Series & shunt impedances Z_1 & Z_2 satisfy the relationship → $Z_1 \times Z_2 = R_0^2$ where R_0 is a real constant or Constant K section.

R_0 is real resistance which is frequency independent, called as design Impedance of the section.

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

For constant-K section:

$$Z_{0\pi} = \frac{R_0^2}{Z_{0T}} \quad \text{where } R_0^2 = Z_1 \times Z_2$$

These constant-K section filters of π or T are called as prototype section.

Constant-K Low pass Filter:-

The low pass filter consists of inductive series arm & capacitive shunt arm.

ⓐ Design impedance (R_0):-

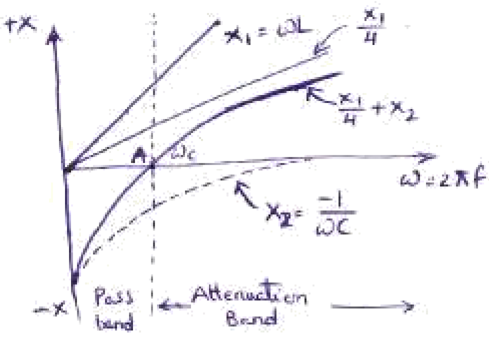
For series arm, $Z_1 = j\omega L$

For shunt arm, $Z_2 = \frac{-j}{\omega C}$

$\Rightarrow Z_1 \cdot Z_2 = j\omega L \times \frac{-j}{\omega C} = \frac{L}{C}$ which is real & constant

$\Rightarrow R_0^2 = \frac{L}{C} \Rightarrow R_0 = \sqrt{\frac{L}{C}}$

ⓑ Reactance curves & cut off frequency expression:-



f_c for T & π will be the same.

$Z_1 = j\omega L$; $Z_2 = \frac{-j}{\omega C}$

$\Rightarrow Z_1 = \omega L$; $Z_2 = \frac{-1}{\omega C}$

From the curves, point A makes the cutoff frequency where $\omega = \omega_c$; curve for $\left(\frac{x_1}{4} + x_2\right)$ crosses freq axis

$\therefore \frac{\omega_c L}{4} - \frac{1}{\omega_c C} = 0$

$\Rightarrow \frac{\omega_c L}{4} = \frac{1}{\omega_c C}$

$\omega_c^2 = \frac{4}{LC} \Rightarrow \omega_c = \frac{2}{\sqrt{LC}}$

ie., $f_c = \frac{1}{\pi\sqrt{LC}}$

(c) Variation of Z_{0T} and $Z_{0\pi}$ with frequency:-

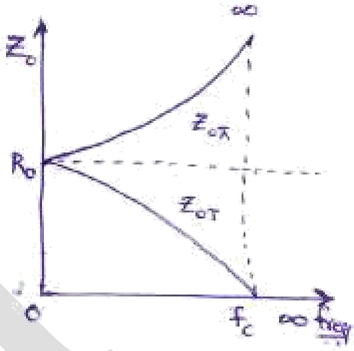
Consider $Z_{0T} = R_0\sqrt{1-\frac{\omega^2 LC}{4}}$ But we know $\omega_c^2 = \frac{4}{LC}$

$\Rightarrow Z_{0T} = R_0\sqrt{1-\frac{\omega^2}{\omega_c^2}}$ (or) $R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}$

III^{ly} $Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$

$\Rightarrow$ As $f\uparrow$ from $0\rightarrow f_c$, Z_{0T} decreases from $R_0\rightarrow 0$ in passband.

As $f\uparrow$ from $0-f_c$, $Z_{0\pi}$ increases from $R_0\rightarrow\infty$ in passband



(d) Variation of Attenuation Constant α with frequency:-

we have $\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ For L.P.F, $Z_1 = j\omega L$, $Z_2 = \frac{-j}{\omega C}$

$\Rightarrow \sinh\frac{\gamma}{2} = \sqrt{\frac{+j\omega L}{4\left(\frac{-j}{\omega C}\right)}} = \sqrt{\frac{-\omega^2 LC}{4}} = j\omega\frac{\sqrt{LC}}{2}$

But we already know for L.P.F; $\omega_c = \frac{2}{\sqrt{LC}}$ $\therefore \frac{1}{\omega_c} = \frac{\sqrt{LC}}{2}$

$\therefore \sinh\frac{\gamma}{2} = j\left(\frac{\omega}{\omega_c}\right) = j\left(\frac{f}{f_c}\right)$

$\Rightarrow \sinh\left(\frac{\alpha}{2}+j\frac{\beta}{2}\right) = j\left(\frac{f}{f_c}\right)$

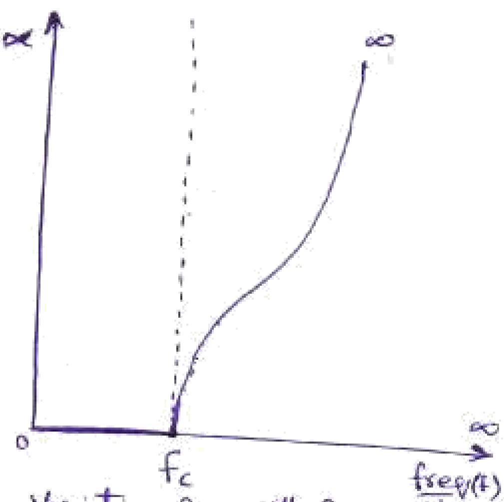
$\Rightarrow \sinh\left(\frac{\alpha}{2}\right)\cdot\cos\left(\frac{\beta}{2}\right) + j\cosh\left(\frac{\alpha}{2}\right)\cdot\sin\left(\frac{\beta}{2}\right) = j\left(\frac{f}{f_c}\right)$

In pass band $\alpha = 0$; & in stop band $\beta = \pi^c$

$\therefore \cos\frac{\beta}{2} = \cos\frac{\pi}{2} = 0$ & $\sin\frac{\beta}{2} = \sin\frac{\pi}{2} = 1$

$\Rightarrow j\cosh\left(\frac{\alpha}{2}\right) = j\left(\frac{f}{f_c}\right)$

$$\boxed{\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)}$$



Variation of α with freq

In stop band, as frequency increases above f_c Attenuation also increases.

(e) Variation of phase constant β with frequency:-

we have

$$\sinh\frac{\gamma}{2} = j\left(\frac{f}{f_c}\right)$$

ie., $\sinh\left(\frac{\alpha}{2}\right)\cdot\cos\left(\frac{\beta}{2}\right) + j\cosh\left(\frac{\alpha}{2}\right)\cdot\sin\left(\frac{\beta}{2}\right) = j\left(\frac{f}{f_c}\right)$

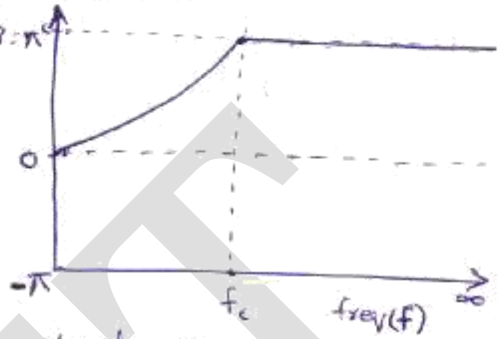
In stopband $\beta = \pi^c$. so β to be calculated in passband when $\alpha = 0$

$$\sinh\frac{\alpha}{2} = \sinh 0 = 0 \;\; \& \;\; \cosh\frac{\alpha}{2} = \cosh 0 = 1$$

$$\Rightarrow j\sin\frac{\beta}{2} = j\left(\frac{f}{f_c}\right)$$

$$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)$$

As frequency increases from $0 \rightarrow f_c$, β also increases from $0 \rightarrow \pi$ radians.



Variation of β with frequency

(f) Design Equations of Prototype Low Pass Filter:-

R_0, design impedance & f_c, cut off frequency can be given in terms of L & C.

$$R_0 = \sqrt{\frac{L}{C}} \quad \& \quad f_c = \frac{1}{\pi\sqrt{LC}}$$

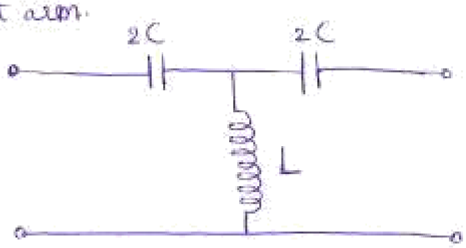
Divide R_0 by f_c,
$$\frac{R_0}{f_c} = \frac{\sqrt{\frac{L}{C}}}{\frac{1}{\pi\sqrt{LC}}} \Rightarrow \boxed{L = \frac{R_0}{(\pi f_c)}} \rightarrow (1)$$

Multiplying R_0 & f_c,
$$\boxed{C = \frac{1}{(\pi f_c)\cdot R_0}} \rightarrow (2)$$

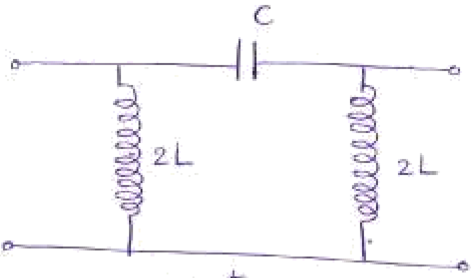
These two equations are called as Design Equations for prototype Low pass filter

Constant-K High Pass filter:-

The high pass filter network consists of capacitive series arm & inductive shunt arm.



T-Section



π section

a) Design Impedance (R_0)

Series arm, Total impedance, $Z_1 = \frac{-j}{\omega C}$

Shunt arm, " " $Z_2 = j\omega L$

$Z_1 \cdot Z_2 = \left(\frac{-j}{\omega C}\right) \cdot j\omega L = \frac{L}{C}$ real & Constant

∴ Given π & T section are constant-K sections.

$$R_0^2 = Z_1 \cdot Z_2 = \frac{L}{C} \Rightarrow R_0 = \sqrt{\frac{L}{C}}$$

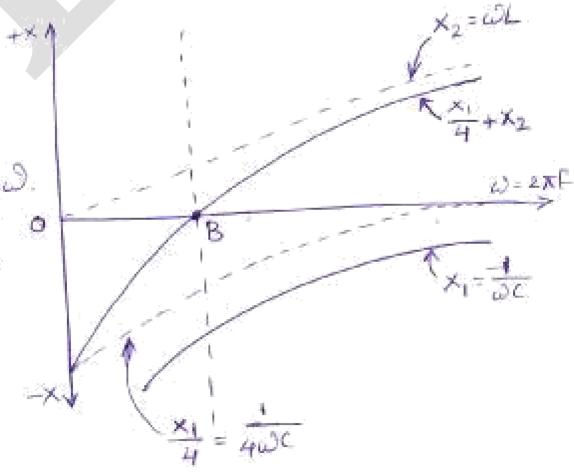
b) Reactance curves & Expression for cutoff frequency:-

we know f_c for T & π-sections is same.

$Z_1 = \frac{-j}{\omega C}$; $Z_2 = (j\omega L)$

$X_1 = \frac{-1}{\omega C}$; $X_2 = \omega L$

Hence $\frac{X_1}{4} + X_2 = \frac{-1}{4\omega C} + \omega L = \omega L - \frac{1}{4\omega C}$



Hence Reactance curves have positive slope as all curves slope upwards to the right side with increasing ω.

Here the curves are on same side of horizontal axis upto the point B, gives a stop band.

For frequencies above point B, the curves are on opposite side of the axis, giving Pass band.

Thus Point B gives cut-off frequency as $\omega = \omega_c$.

$$\Rightarrow \omega_c L - \frac{1}{4\omega_c C} = 0$$

$$\omega_c L = \frac{1}{4\omega_c C}$$

$$\omega_c^2 = \frac{1}{4LC}$$

$$\omega_c = \frac{1}{2\sqrt{LC}} \quad (\text{or}) \quad \boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

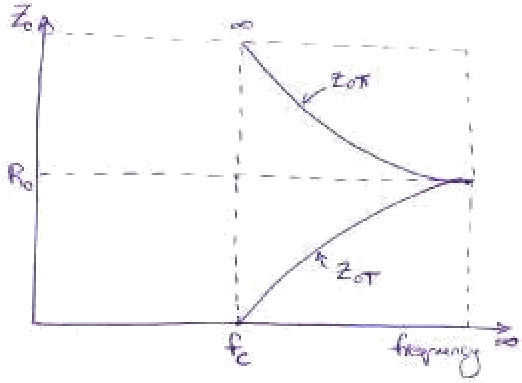
$$\therefore Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}} = \sqrt{\frac{L}{C}} \cdot \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$\Rightarrow \boxed{Z_{0T} = R_0 \sqrt{1 - \frac{1}{4\omega^2 L C}}}$$

(c) Variation of Z_{0T} & $Z_{0\pi}$ with frequency :-

consider $Z_{0T} = R_0\sqrt{1-\frac{1}{4\omega^2 LC}}$

$\Rightarrow Z_{0T} = R_0\sqrt{1-\frac{\omega_c^2}{\omega^2}}$ (or) $Z_{0T} = R_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}$



III'ly $Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

Hence it is clear that frequency increases from f_c to ∞ in pass band, Z_{0T} also increases from 0 → R_0.

Also frequency increses from f_c to ∞ in pass band & $Z_{0\pi}$ decreases from ∞ to R_0.

(d) Variation of Attenuation constant (α) with frequency :-

we have $\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ | For H.P.F, $Z_1 = \frac{-j}{\omega C}$ & $Z_2 = j\omega L$

$\Rightarrow \sinh\frac{\gamma}{2} = \sqrt{\frac{-j}{(\omega C)(4)(j\omega L)}} = \sqrt{\frac{-1}{4\omega^2 LC}} = j\sqrt{\frac{1}{\omega^2 4LC}} = j\frac{1}{\omega(2\sqrt{LC})}$

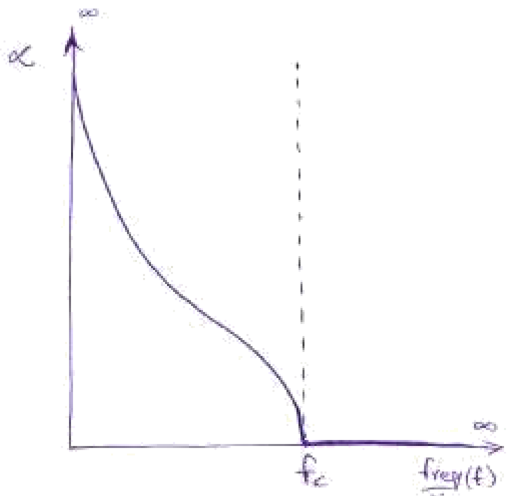
for high pass filter, we know, $\omega_c = \frac{1}{2\sqrt{LC}}$

$\Rightarrow \sinh\frac{\gamma}{2} = j\left(\frac{\omega_c}{\omega}\right) = j\left(\frac{f_c}{f}\right)$

$\Rightarrow \sinh\left(\frac{\alpha}{2}+j\frac{\beta}{2}\right) = j\left(\frac{f_c}{f}\right)$

$\Rightarrow \sinh\left(\frac{\alpha}{2}\right)\cdot\cosh j\left(\frac{\beta}{2}\right) + \cosh\left(\frac{\alpha}{2}\right)\cdot\sinh j\left(\frac{\beta}{2}\right) = j\left(\frac{f_c}{f}\right)$

$\Rightarrow \sinh\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) + j\cosh\left(\frac{\alpha}{2}\right)\cdot\sin\left(\frac{\beta}{2}\right) = j\left(\frac{f_c}{f}\right)$



$\alpha = 0$ for passband. In stop band $\beta = \pi^c$ & α to be found.

$\cos\frac{\beta}{2} = \cosh\frac{\pi}{2} = 0$; $\sin\frac{\beta}{2} = \sin\frac{\pi}{2} = 1$

$\Rightarrow j\cosh\left(\frac{\alpha}{2}\right) = j\left(\frac{f_c}{f}\right)$

$$\boxed{\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)}$$

In stop band, as frequency increases from 0 → f_c, α decreases from ∞ to 0.

(e) Variation of phase constant β with frequency:-

we have $\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha+j\beta}{2}\right) = j\left(\frac{f_c}{f}\right)$

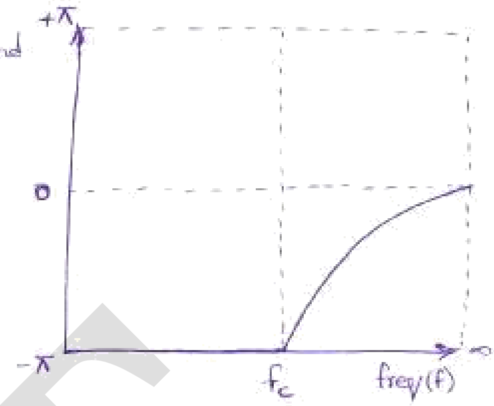
$\Rightarrow \sinh\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) + j\cosh\left(\frac{\alpha}{2}\right)\cdot\sin\left(\frac{\beta}{2}\right) = j\left(\frac{f_c}{f}\right)$

In stop band $\beta = \pi^c$; β to be calculated for passband when $\alpha = 0$

$\therefore \sinh\left(\frac{\alpha}{2}\right) = \sinh 0 = 0$; $\cosh\left(\frac{\alpha}{2}\right) = \cosh 0 = 1$

$\Rightarrow j\sin\left(\frac{\beta}{2}\right) = j\left(\frac{f_c}{f}\right)$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)}$$



As frequency increases from f_c to ∞, β decreases from $-\pi$ to 0.

(f) Design Equations of prototype High Pass filter:-

R_0, design impedance & f_c, cutoff frequency for high pass filter interms of L & C

$R_0 = \sqrt{\frac{L}{C}}$ & $f_c = \frac{1}{4\pi\sqrt{LC}}$

Dividing R_0 by f_c,

$\frac{R_0}{f_c} = \frac{\sqrt{\frac{L}{C}}}{\frac{1}{4\pi\sqrt{LC}}} \Rightarrow \boxed{L = \frac{R_0}{(4\pi f_c)} \rightarrow ①}$

Multiplying R_0 & f_c,

$$\boxed{C = \frac{1}{(4\pi f_c)\, R_0} \rightarrow ②}$$

Equations ① & ② are called Design Equations of Prototype High pass filter Sections

Disadvantages of prototype Filter Section:-

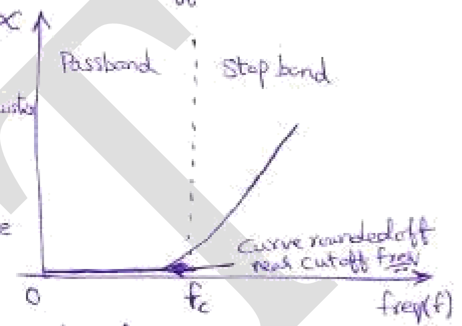
① Ideally, the attenuation should change sharply in the attenuation band. But in all prototype filter sections, attenuation gradually changes in stopband. Hence frequencies near cutoff frequency are passed through the filter.

② In pass band, o/p of filter should remain constant. ie., Z_0 should be constant. But Z_0 varies with frequency from value R_0, throughout passband. Hence filter cannot be terminated properly.

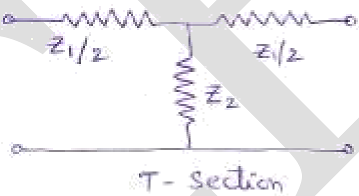
M-Derived Filter Section :-

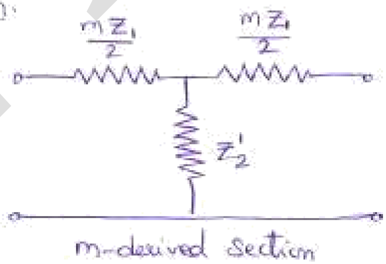
The first disadvantage of prototype filter sections can be overcome by connecting two or more prototype sections of same type in cascade. In such cascade connection, attenuation to the frequencies in passband remains zero ideally, but attenuation to the frequencies in entire attenuation band considerably increases. ie, attenuation band doubles giving sharper cutoff characteristics than a single section. But due to the resistance used in cascade connection, the attenuation in passband slightly increases, instead of being zero. (curve becomes rounded off at f_c in Passband)

So, it is necessary to design a new section with same cut off frequency, but different Attenuation characteristics in the attenuation band. Also to maintain same, f_c both the sections must have same Z_0. It is possible to derive new section from prototype constant-K section. Thus new section derived from is called as m-derived section.



Tsection	m-section
$Z_1/2$	$mZ_1/2$
Z_2	Z_2'





For prototype section, $Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$

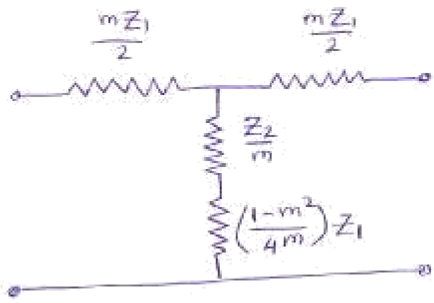
For m-derived section, $Z_{0T} = \sqrt{\frac{(mZ_1)^2}{4} + (mZ_1)(Z_2')}$

For same value of Z_{0T}, equating both equations,

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + mZ_1 Z_2'$$

$$mZ_1 Z_2' = \frac{Z_1^2}{4} - \frac{m^2 Z_1^2}{4} + Z_1 Z_2$$

$$\Rightarrow Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}$$

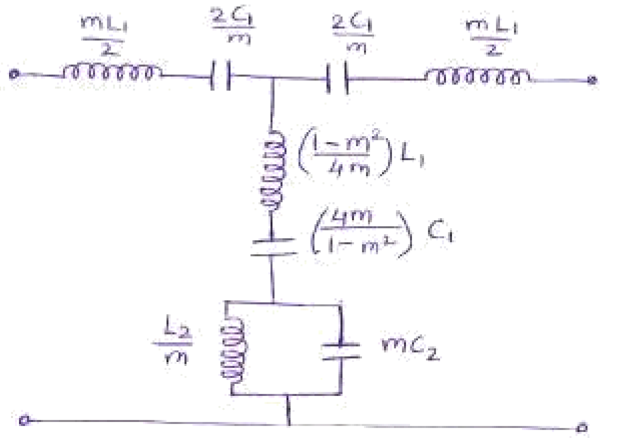


ie, shunt arm is series of two impedances.

In m-derived section filters, Attenuation characteristics can be improved in stop band by using series resonant circuit form in the shunt arm of m-derived T-Section.

m derived Band Pass Filter :-

We can obtain m-derived band pass filter as shown in the figure. The T-section in each case will have shunt impedance $\frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right) Z_1$ where Z_1 & Z_2 are impedances of prototype section.



m-derived band pass filter section

m-derived bandpass filter can be obtained by

$$\frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right) Z_1 = 0$$

If we substitute the values of Z_1 & Z_2 in above equation, we get two value of frequencies. These frequencies are frequencies of attenuation $f_{1\infty}$ and $f_{2\infty}$.

If the frequency of resonance is f_0, then relationship between f_c & $f_{1\infty}$, $f_{2\infty}$ is

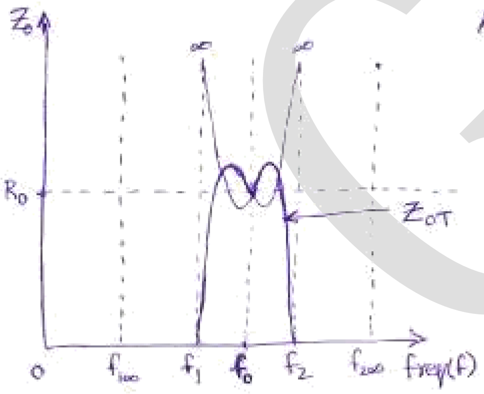
$$f_0 = \sqrt{(f_{1\infty})(f_{2\infty})} = \sqrt{(f_1)(f_2)}$$

Also we can write

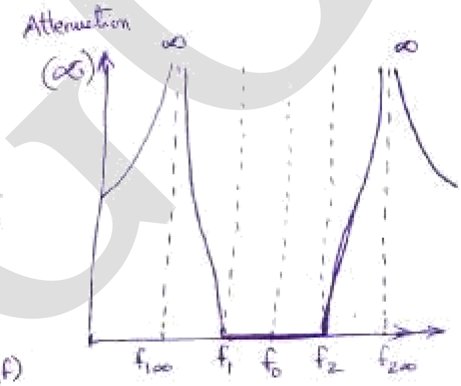
$$(f_{2\infty} - f_{1\infty}) = \frac{f_2 - f_1}{\sqrt{1-m^2}}$$

$$\Rightarrow \sqrt{1-m^2} = \frac{(f_2 - f_1)}{(f_{2\infty} - f_{1\infty})} \Rightarrow m^2 = 1 - \left[\frac{f_2 - f_1}{f_{2\infty} - f_{1\infty}}\right]^2 \Rightarrow m = \sqrt{1 - \left(\frac{f_2 - f_1}{f_{2\infty} - f_{1\infty}}\right)^2}$$

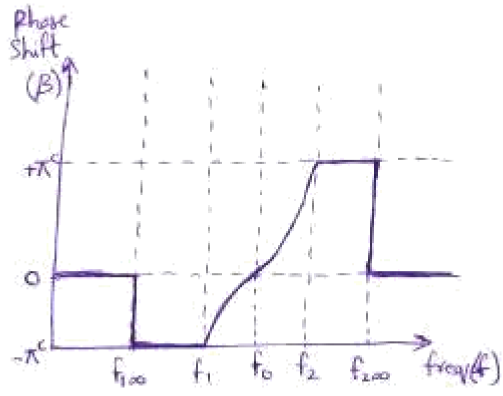
Variation of Z_0, α and β with frequency :-



ⓐ Variation of Z_0 with frequency



ⓑ Variation of α with frequency



ⓒ Variation of β with frequency

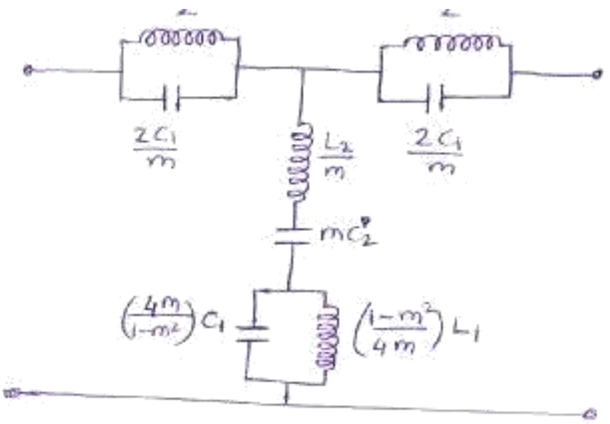
m-derived band Elimination filter:-

The relationship of frequency of attenuation ($f_{1\infty}, f_{2\infty}$) and cut off frequencies (f_1, f_2) is

$$(f_{2\infty} - f_{1\infty}) = (f_2 - f_1)\sqrt{1-m^2}$$

If 'f_0' is the frequency of resonance, then

$$f_0 = \sqrt{(f_1)(f_2)} = \sqrt{(f_{1\infty})(f_{2\infty})}$$

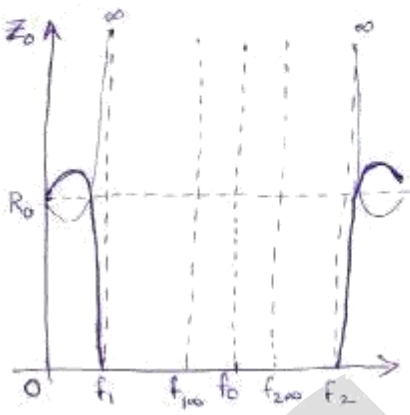


m-derived band Elimination filter.

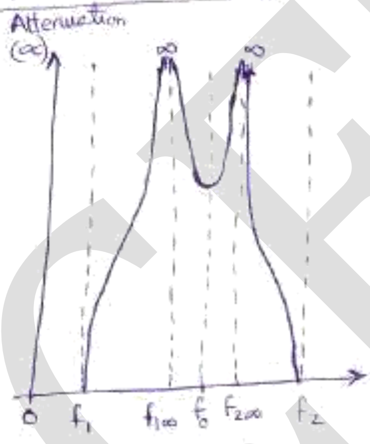
Rewriting earlier eq,

$$\sqrt{1-m^2} = \frac{(f_{2\infty} - f_{1\infty})}{(f_2 - f_1)} \Rightarrow m^2 = 1 - \left(\frac{f_{2\infty} - f_{1\infty}}{f_2 - f_1}\right)^2 \Rightarrow \boxed{m = \sqrt{1 - \left(\frac{f_{2\infty} - f_{1\infty}}{f_2 - f_1}\right)^2}}$$

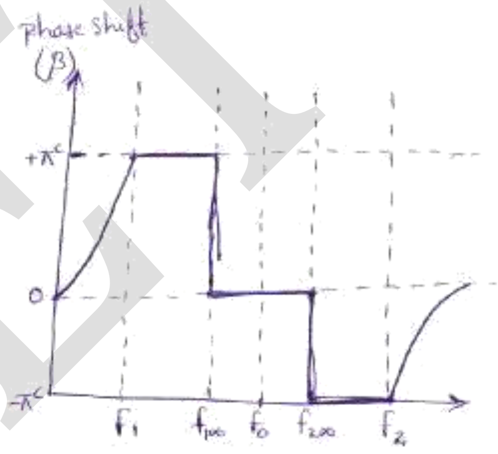
Variation of Z_0, α and β with frequency:-



Variation of Z_0 with frequency



Variation of α with frequency



Variation of β with frequency.

Unit-4 Symmetrical Attenuators

Generally in power transmission equipments, many times it is required to supress (or) attenuate the levels of currents & voltages at certain points. To fulfil the need of attenuation, a four terminal resistive network called attenuator is used. They are designed to provide required amount of attenuation between the input and output terminals. As these networks are resistive, so all frequencies are attenuated by same degree of value preventing attenuation distortion. There will be no phase shift in such networks, hence $\beta = 0$ & propagation constant (γ) will be equal to attenuation constant (α). Its units are neper (or) decibel.

Attenuators are either symmetrical (or) asymmetrical networks. They can also be of fixed value (or) adjustable value type. Fixed attenuators providing constant attenuation are called pads. Variable attenuators are used generally in Radio broadcasting stations as volume controls.

Power, Voltage and Current Ratios:-

In line communication when A.C power from sending end to receiving end is considered, various elements in the system introduce gains (or) losses in power.

Consider a 4-terminal network with P_1 & P_2 as I/p & O/p Powers

Power ratio $\frac{P_1}{P_2} = M$ $M > 1 \rightarrow$ loss of power; $M < 1 \rightarrow$ Power gain

For n- network connected in cascade (or) tandem, overall power ratio is

$$\frac{P_1}{P_n} = \frac{P_1}{P_2} \times \frac{P_2}{P_3} \times \frac{P_3}{P_4} \times \cdots \times \frac{P_{n-1}}{P_n}$$

$$M = M_1 \times M_2 \times M_3 \times \cdots \times M_{n-1}$$

Power ratio in "BELL" (Logarithmic unit)

$$\frac{P_1}{P_n} = \log_{10} \left|\frac{P_1}{P_n}\right| \text{ Bell}$$

$\because$ "BELL" is too large value, often "decibel" is used

$\therefore D = \frac{P_1}{P_2} = 10 \log_{10} \left|\frac{P_1}{P_2}\right|$ decibel

for $M > 1 \Rightarrow D$ is +ve $\Rightarrow$ Powerloss

for $M < 1 \Rightarrow D$ is −ve $\Rightarrow$ Powergain

Also $\frac{P_1}{P_2} = \text{Antilog} \left|\frac{D}{10}\right|$

we know $P_1 = E_1 \cdot I_1 = (R I_1) \cdot I_1 = R \cdot I_1^2$

$= E_1 \cdot \left(\frac{E_1}{R}\right) = \frac{E_1^2}{R}$

$P_2 = E_2 I_2 = (R I_2) \cdot I_2 = R \cdot I_2^2$

$= E_2 \cdot \left(\frac{E_2}{R}\right) = \frac{E_2^2}{R}$

∴ Power ratio given by

$$\frac{P_1}{P_2} = \frac{E_1 I_1}{E_2 I_2} = \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{E_1}{E_2}\right)^2$$

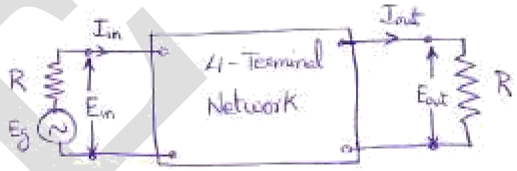
In decibel,

⟹ $D = 10 \log_{10}\left|\frac{P_1}{P_2}\right| = 10 \log_{10}\left|\frac{I_1}{I_2}\right|^2 = 20 \log_{10}\left|\frac{I_1}{I_2}\right|$ &

$D = 10 \log_{10}\left|\frac{P_1}{P_2}\right| = 10 \log_{10}\left|\frac{E_1}{E_2}\right|^2 = 20 \log_{10}\left|\frac{E_1}{E_2}\right|$

Expression for Attenuation :-

Attenuation is defined as loss of power in transmission line (or) an electrical network. It is expressed in neper(N) (or) decibel (dB).



Attenuation (in decibel) $= 10 \log_{10}\left|\frac{P_{in}}{P_{out}}\right|$

III^{ly} Attenuation (in neper) $= \ln\left|\frac{I_{in}}{I_{out}}\right|$ for current

when $R_{in} = R_{out} = R$,

Attenuation (in decibel) $= 10 \log_{10}\left|\frac{P_{in}}{P_{out}}\right|$

$= 10 \log_{10}\left|\frac{I_{in}^2 R}{I_{out}^2 R}\right|$

$= 20 \log_{10}\left|\frac{I_{in}}{I_{out}}\right| = 20 \log_{10}\left|\frac{E_{in}}{E_{out}}\right|$

Attenuation (in neper) $= \ln\left|\frac{I_{in}}{I_{out}}\right| = \ln\left|\frac{E_{in}}{E_{out}}\right| = \frac{1}{2}\ln\left|\frac{P_{in}}{P_{out}}\right|$

Relation of neper & Decibel:-

> Attenuation in dB = 8.686 × Attenuation in neper &
> Attenuation in neper = 0.1151 × Attenuation in dB

Attenuator Networks:-

An attenuator network must fulfil following conditions:

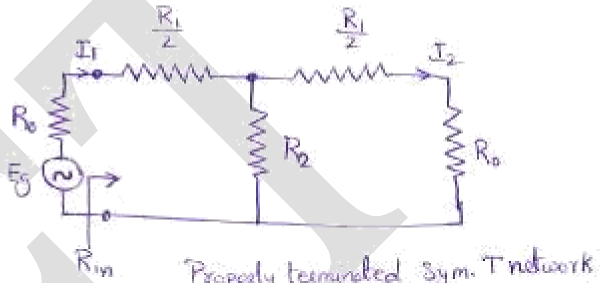
1. It must give correct input impedance
2. It must give correct output impedance &
3. It should provide specified attenuation.

$D = 10 \log_{10}\left|\frac{P_{in}}{P_{out}}\right|$ Where D is the attenuation in decibel.

Also $D = 20 \log_{10}\sqrt{\frac{P_{in}}{P_{out}}} = 20 \log_{10} N$, where N is attenuation in neper

$\Rightarrow N = \text{Antilog}_{10}\left[\frac{D}{20}\right]$ neper

① Symmetrical T-Type Attenuator:-



Properly terminated sym. T network

Method 1:-

For symmetrical T-Network, series & shunt arm impedances are given by

$$\frac{Z_1}{2} = Z_0 \tanh\frac{\gamma}{2} \quad \& \quad Z_2 = \frac{Z_0}{\sinh\gamma}$$

for symmetrical T-attenuator, we have

$$\frac{Z_1}{2} = \frac{R_1}{2} \; ; \; Z_2 = R_2 \; ; \; Z_0 = R_0 \; \& \; \gamma = \alpha$$

Hence $\frac{R_1}{2} = R_0 \tanh\left(\frac{\alpha}{2}\right)$ ① ; $R_2 = \frac{R_0}{\sinh\alpha}$ ②

$$\Rightarrow \frac{R_1}{2} = R_0\left[\frac{e^{\frac{\alpha}{2}} - e^{-\frac{\alpha}{2}}}{e^{\frac{\alpha}{2}} + e^{-\frac{\alpha}{2}}}\right]$$

multiply num & den by $e^{\frac{\alpha}{2}}$ on RHS:

$$\frac{R_1}{2} = R_0\left[\frac{e^{\alpha}-1}{e^{\alpha}+1}\right]$$

But $e^{\alpha} = \frac{I_1}{I_2} = N$

$$\Rightarrow \boxed{\frac{R_1}{2} = R_0\left[\frac{N-1}{N+1}\right]} \longrightarrow Ⓐ$$

$$R_2 = \frac{R_0}{\sinh\alpha}$$

$$\Rightarrow \sinh\alpha = \frac{R_0}{R_2}$$

$$\frac{e^{\alpha} - e^{-\alpha}}{2} = \frac{R_0}{R_2}$$

$$\frac{N - \frac{1}{N}}{2} = \frac{R_0}{R_2} \qquad \because e^{\alpha} = N \;\&\; e^{-\alpha} = \frac{1}{N}$$

$$\frac{N^2-1}{2N} = \frac{R_0}{R_2}$$

$$\boxed{R_2 = R_0\left[\frac{2N}{N^2-1}\right]} \longrightarrow Ⓑ$$

Equation Ⓐ & Ⓑ are called design equations of Symmetrical T-type Attenuator.

Method 2 :-

From the figure,

by using current divider rule,

$$I_2 = I_1 \left[\frac{R_2}{R_2 + \left(R_0 + \frac{R_1}{2}\right)} \right]$$

for symmetrical Networks,

$$N = \frac{I_1}{I_2} = \frac{R_0 + R_2 + \frac{R_1}{2}}{R_2}$$

for properly terminated network,

$$R_{in} = R_0 = \left[\left(R_0 + \frac{R_1}{2}\right) \parallel R_2 \right] + \frac{R_1}{2}$$

$$R_0 = \frac{R_2 \left(R_0 + \frac{R_1}{2}\right)}{R_0 + R_2 + \frac{R_1}{2}} + \frac{R_1}{2}$$

$$\Rightarrow R_0 = \frac{R_0 + \frac{R_1}{2}}{N} + \frac{R_1}{2}$$

$$N R_0 = R_0 + \frac{R_1}{2} + N \frac{R_1}{2}$$

$$R_0 (N-1) = \frac{R_1}{2} (N+1)$$

$$\frac{R_1}{2} = R_0 \left(\frac{N-1}{N+1}\right) \longrightarrow (A)$$

$$N R_2 = R_0 + R_2 + \frac{R_1}{2}$$

$$R_2 (N-1) = R_0 + R_0 \left(\frac{N-1}{N+1}\right)$$

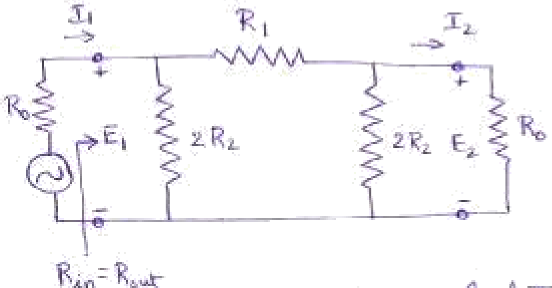
$$R_2 (N^2 - 1^2) = R_0 (N+1) + R_0 (N-1)$$

$$R_2 = R_0 \left[\frac{2N}{N^2 - 1} \right] \longrightarrow (B)$$

Equations (A) & (B) are called as Design Equations of Symmetrical T-Attenuator.

② Symmetrical π type Attenuator :-

Consider a properly terminated π network as shown in the figure



Properly terminated Symmetrical π Attenuator.

Method 1 :-

For a Symmetrical π Network, Series & Shunt arm impedances are given by

$$Z_1 = Z_0 \sinh \gamma \quad \text{and} \quad 2Z_2 = \frac{Z_0}{\tanh \frac{\gamma}{2}}$$

But for the network we have, $Z_1 = R_1$, $2Z_2 = 2R_2$; $Z_0 = R_0$ and $\gamma = \alpha$

$$\Rightarrow R_1 = R_0 \sinh \alpha \quad \& \quad 2R_2 = \frac{R_0}{\tanh \frac{\alpha}{2}}$$

$$\Rightarrow R_1 = R_0 \left[\frac{e^{\alpha} - e^{-\alpha}}{2}\right]$$

But $N = \frac{I_1}{I_2} = e^{\alpha}$

$$\Rightarrow R_1 = R_0 \left[\frac{N - \frac{1}{N}}{2}\right] = \frac{R_0}{2}\left[\frac{N^2 - 1}{N}\right]$$

$$\therefore R_1 = R_0 \left[\frac{N^2 - 1}{2N}\right] \rightarrow ①$$

$$2R_2 = R_0 \left[\frac{1}{\tanh e^{\alpha/2}}\right] = R_0 \left[\frac{1}{\frac{e^{\alpha/2} - e^{-\alpha/2}}{e^{\alpha/2} + e^{-\alpha/2}}}\right]$$

$$2R_2 = R_0 \left[\frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}}\right]$$

multiplying num & Den by $e^{\alpha/2}$ on RHS of expression

$$2R_2 = R_0 \left[\frac{e^{\alpha} + 1}{e^{\alpha} - 1}\right]$$

$$2R_2 = R_0 \left[\frac{N + 1}{N - 1}\right] \rightarrow ②$$

Equations ① & ② are the design equations of symmetrical π attenuator.

Method 2 :- From the figure, for symmetrical π network,

$$Z_0 = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}} \quad \& \quad e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_0}$$

For π attenuator, we have

$$R_0 = \frac{R_1 R_2}{\sqrt{\frac{R_1^2}{4} + R_1 R_2}} \quad \& \quad e^{\alpha} = N = 1 + \frac{R_1}{2R_2} + \frac{R_1}{R_0}$$

$$\Rightarrow R_0^2 = \frac{R_1^2 R_2^2}{\frac{R_1^2}{4} + R_1 R_2}$$

$$\frac{R_1^2}{4} + R_1 R_2 = \frac{R_1^2 R_2^2}{R_0^2} = \left(\frac{R_1 R_2}{R_0}\right)^2$$

$$R_1^2 \left[\frac{1}{4} - \frac{R_2^2}{R_0^2}\right] + R_1 R_2 = 0$$

$$R_1 \left[\frac{1}{4} - \frac{R_2^2}{R_0^2}\right] + R_2 = 0$$

$$R_1 \left[\frac{1}{4} - \frac{R_2^2}{R_0^2}\right] = -R_2$$

$$R_1 = \frac{-4R_2 R_0^2}{R_0^2 - 4R_2^2} \Rightarrow R_1 = \frac{4R_2 R_0^2}{4R_2^2 - R_0^2} \quad ①$$

Substituting the value of R_1 in above equation,

$$N = 1 + \frac{4R_2 R_0^2}{(4R_2^2 - R_0^2)}\left[\frac{R_0 + 2R_2}{2R_2 R_0}\right]$$

$$N = 1 + \frac{2R_0 (R_0 + 2R_2)}{(2R_2 - R_0)(2R_2 + R_0)} = 1 + \frac{2R_0}{(2R_2 - R_0)}$$

$$\Rightarrow (N - 1)(2R_2 - R_0) = 2R_0$$

$$\Rightarrow 2R_2 N - 2R_2 - R_0 N + R_0 = 2R_0$$

$$2R_2 (N - 1) = R_0 (N + 1)$$

$$2R_2 = R_0 \left[\frac{N + 1}{N - 1}\right] \rightarrow ②$$

By substituting Eq ② in Eq ① of e^{α}

$$N = 1 + \frac{R_1}{R_0\left[\frac{N+1}{N-1}\right]} + \frac{R_1}{R_0}$$

$$(N-1) = R_1\left[\frac{N-1}{R_0(N+1)} + \frac{1}{R_0}\right]$$

$$(N-1) = R_1\left[\frac{N-1+N+1}{R_0(N+1)}\right] = R_1\left[\frac{2N}{R_0(N+1)}\right]$$

$$\Rightarrow \boxed{R_1 = R_0\left[\frac{N^2-1}{2N}\right]} \rightarrow ③$$

Eq ② & ③ are called the design equations of symmetrical π attenuator.

Problem ① Design sym π attenuator with 20dB attenuation and 600Ω design impedance.

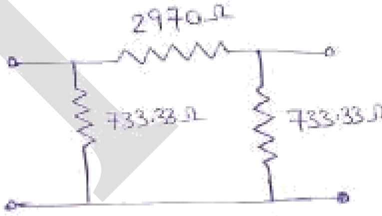
Sol:- For π Network, $D = 20dB$ & $R_0 = 600\Omega$

$$\therefore N = \text{Antilog}_{10}\left(\frac{D}{20}\right) = \text{Antilog}_{10}\left(\frac{20}{20}\right) = 10$$

For π attenuators,

$$R_1 = R_0\left[\frac{N^2-1}{2N}\right] = 600\left[\frac{100-1}{20}\right] = 2970\Omega$$

$$2R_2 = R_0\left[\frac{N+1}{N-1}\right] = 600\left[\frac{10+1}{10-1}\right] = 733.33\Omega$$



Sym. π type Attenuator

② Design a π-type attenuator to provide attenuation of 10dB and working with characteristic impedance of 600Ω. Mention some applications of Attenuators.

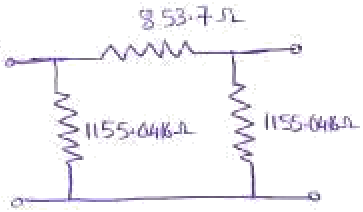
Sol:- For π Network $D = 10dB$, $R_0 = 600\Omega$

$$N = \text{Antilog}\left(\frac{D}{20}\right) = \text{Antilog}\left(\frac{10}{20}\right) = 3.162$$

For π-Attenuator;

$$R_1 = R_0\left(\frac{N^2-1}{2N}\right) = 600\left(\frac{(3.162)^2-1}{2(3.162)}\right) = 853.7\Omega$$

$$2R = R_0\left[\frac{N+1}{N-1}\right] = 600\left[\frac{3.162+1}{3.162-1}\right] = 1155.0416\Omega$$



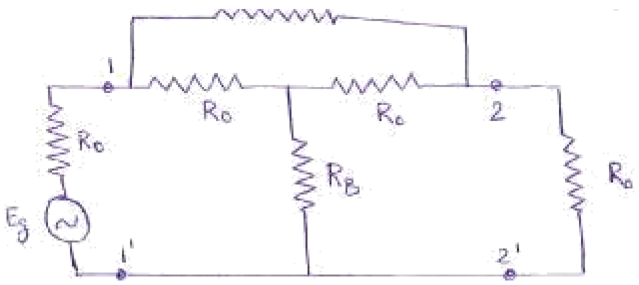
Attenuators may be either symmetrical (or) asymmetrical.

They can also be either fixed value (or) variable value type.

Fixed value attenuators provide constant attenuation and called as Pads.

Variable value attenuators are used in Radio broadcasting stations as volume controls.

③ Bridged T type Attenuator :-



Bridged T attenuator

Method 1 :-

Generally in bridged T attenuator, each of the series arm is made equal to Z_0.

$$Z_0 = \sqrt{Z_2 \cdot Z_3} \quad \& \quad \gamma = \ln\left[1+\frac{Z_3}{Z_0}\right] = \ln\left[1+\frac{Z_0}{Z_2}\right]$$

we have,

$$Z_2 = R_B ;\ Z_3 = R_A ;\ Z_0 = R_0 \ \&\ \gamma = \alpha$$

$$\Rightarrow R_0 = \sqrt{R_A \cdot R_B} \longrightarrow ① \quad \& \quad \alpha = \ln\left[1+\frac{R_A}{R_0}\right] = \ln\left[1+\frac{R_0}{R_B}\right] \longrightarrow ②$$

From ② $\quad e^{\alpha} = N = \left[1+\frac{R_A}{R_0}\right] = \left[1+\frac{R_0}{R_B}\right]$

For R_A,

$$N = \left[1+\frac{R_A}{R_0}\right]$$

$$\frac{R_A}{R_0} = (N-1)$$

$$R_A = R_0(N-1) \quad Ⓐ$$

For R_B,

$$N = \left[1+\frac{R_0}{R_B}\right]$$

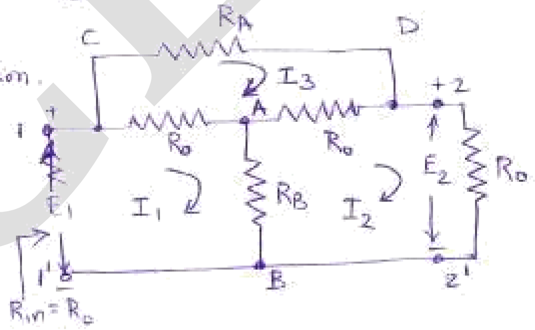
$$\frac{R_0}{R_B} = (N-1)$$

$$R_B = \frac{R_0}{(N-1)} \quad Ⓑ$$

Equation Ⓐ & Ⓑ are design equations of bridged T Attenuator.

Method 2 :-

Assume 3 loop currents in clockwise direction.



Consider path 1 – A – B – 1' – 1, Apply KVL,

$$R_0 I_1 - R_0 I_3 + R_B I_1 - R_B I_2 = E_1$$

$$(R_0 + R_B) I_1 - R_B I_2 - R_0 I_3 = E_1 = R_0 I_1$$

$$\Rightarrow R_B I_1 - R_B I_2 - R_0 I_3 = 0 \longrightarrow ①$$

Consider path A – 2 – 2' – B – A, Apply KVL,

$$R_0 I_2 + R_0 I_2 + R_B I_2 - R_B I_1 - R_0 I_3 = 0$$

$$+ R_B I_1 - (2R_0 + R_B) I_2 + R_0 I_3 = 0 \longrightarrow ②$$

Consider path C – D – 2 – A – 1 – C, Apply KVL,

$$R_A I_3 + R_0 I_3 - R_0 I_2 + R_0 I_3 - R_0 I_1 = 0$$

$$R_0 I_1 + R_0 I_2 - (2R_0 + R_A) I_3 = 0 \longrightarrow ③$$

By adding Equations ① & ②

$$2R_B I_1 - (2R_0 + 2R_B) I_2 = 0$$

$$2R_B I_1 = 2(R_0 + R_B) I_2$$

$$\frac{I_1}{I_2} = N = 1 + \frac{R_0}{R_B}$$

$$R_B = \frac{R_0}{(N-1)} \longrightarrow ④$$

From eq ①,

$$R_B(I_1 - I_2) = R_0 I_3$$

$$I_3 = R_B\left[\frac{I_1 - I_2}{R_0}\right]$$

by substituting value of I_3 in ③

$$R_0 I_1 + R_0 I_2 - (2R_0 + R_A)\left(R_B \frac{(I_1 - I_2)}{R_0}\right) = 0$$

$$R_0 I_1 + R_0 I_2 = (2R_0 + R_A)\left(\frac{R_B (I_1 - I_2)}{R_0}\right)$$

Substitue value of R_B in above equation,

$$R_0 I_1 + R_0 I_2 = (2R_0 + R_A)\left[\frac{R_0}{(N-1)} \cdot \frac{I_1 - I_2}{R_0}\right]$$

$$(R_0 I_1 + R_0 I_2)(N-1) = (2R_0 + R_A)(I_1 - I_2)$$

$$NR_0 I_1 + NR_0 I_2 - R_0 I_1 - R_0 I_2 = 2R_0 I_1 + R_A I_1 - 2R_0 I_2 - R_A I_2$$

$$(NR_0 - R_0 - 2R_0 - R_A) I_1 = I_2(-2R_0 - R_A - NR_0 + R_0)$$

$$N = \frac{I_1}{I_2} = \frac{-(NR_0 + R_A + R_0)}{(NR_0 - 3R_0 - R_A)}$$

$$N^2 R_0 - 3NR_0 - NR_A = -NR_0 - R_A - R_0$$

$$-NR_A + R_A = -N^2 R_0 + 2NR_0 - R_0$$

$$-R_A(N-1) = -R_0(N^2 - 2N + 1)$$

$$R_A = \frac{R_0 (N-1)^2}{(N-1)} \Longrightarrow \boxed{R_A = R_0(N-1)} \longrightarrow ⑤$$

Equations ④ & ⑤ are called design equations of bridged T attenuator.

$$\Rightarrow \frac{I_1}{I_2} = N = \frac{R_0 + R_A}{R_0 - R_A}$$

$$\Rightarrow N(R_0 - R_A) = R_0 + R_A$$

$$\Rightarrow R_A(N+1) = R_0(N-1)$$

$$R_A = R_0\left[\frac{N-1}{N+1}\right] \longrightarrow$$

Now by applying KVL to closed path A-1-2'-2-1'-B-A

$$R_B(I_1 - I) - R_0 I_2 + R_B(I - I_2) - I_1 R_0 = 0$$

$$R_B I_1 - R_B I - R_0 I_2 + R_B I - R_B I_2 - I_1 R_0 = 0$$

$$\Rightarrow I_2(R_0 + R_B) = I_1(R_B - R_0)$$

$$\frac{I_1}{I_2} = N = \frac{R_B + R_0}{R_B - R_0}$$

$$N(R_B - R_0) = R_B + R_0$$

$$R_B(N-1) = R_0(N+1)$$

$$R_B = R_0\left[\frac{N+1}{N-1}\right] \longrightarrow$$

These two Equations are called design Equations of Symmetrical lattice Attenuator.

UNIT-~~5~~6 D.C. Machines (EM-I)

5. DC ~~Motors~~ Generators

A Machine that converts mechanical energy (or Power) into Electrical Energy (or Power) of d.c. Nature is called D.C. Generator. The basic principle of working of a d.c. generator is Faradays laws of Electro Magnetic Induction, which states that, whenever a conductor cuts the magnetic field flux, dynamically induced emf is produced. This emf causes current to flow if the conductor circuit is closed.

The basic essential parts of electrical generator are:

① Magnetic field.

② conductor.

The direction of induced emf depends upon the direction of magnetic field and the direction of motion and is given by flemings right hand rule.

A d.c. Machine that converts electric energy (or Power) into mechanical energy (or Power) is called a d.c. Motor. The d.c. Motor basically works on the principle that when a current carrying conductor placed in a magnetic field, mechanical force acts on the current carrying conductor.

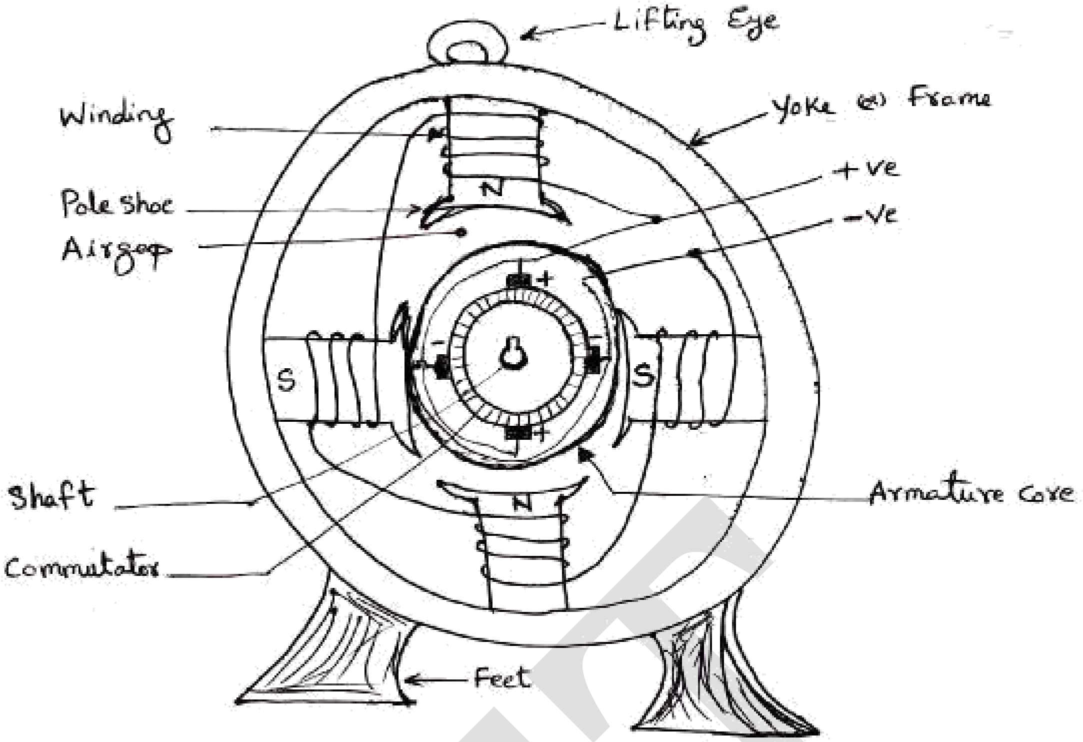
Construction of D.C. Machine:-

A D.C Machine consists of

(i) Magnetic frame (or) Yoke
(ii) Pole cores and Pole shoes.
(iii) Field coils
(iv) Armature core
(v) Armature winding
(vi) Commutator
(vii) Brushes and Bearings.

(i), (ii), (iv) + Airgap → Form Magnetic circuit

(iii), (v), (vi), (vii) → Form Electrical circuit.



Cross-Sectional Area of a D.C. Machine.

① Yoke: The outer frame (or) yoke serves two purposes.

(i) It provides mechanical support to poles & protective cover for whole machine.

(ii) It carries magnetic flux produced by poles.

For small machines yoke is made of cast-iron, but for large machines cast-steel (or) Rolled steel is used.

② Pole core and Pole shoes:-

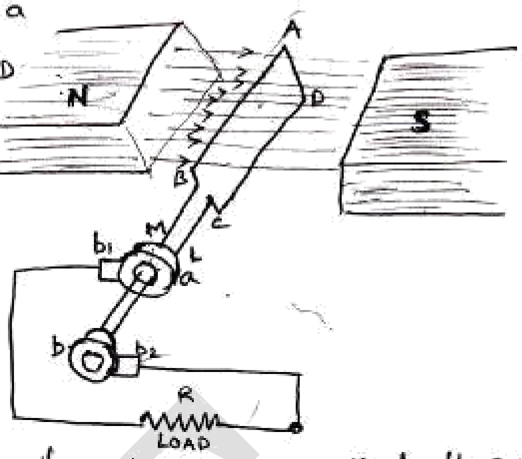
The field magnets consists of pole cores and pole shoes. Pole core is usually of circular section. They are made of cast steel (or) wrought iron laminations and are fixed to the yoke. They carry coils of insulated copper wires carrying the exciting current.

Pole shoes serve two purposes. (i) They spread out the flux in the airgap.
(ii) They support the field coils.

③ Field Coils:- The coils of copper wire wound round the poles are called the field coils (or) pole coils. When current is passed through these coils, they Electromagnetise the poles which produce the necessary flux that is cut by the revolving armature conductors.

Working principle of D.C. Generator :-

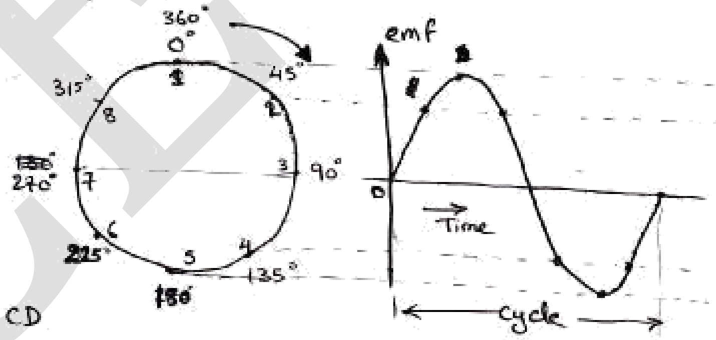
Figure shows the schematic diagram of a simple machine consisting of a coil ABCD moving about its own axis in a magnetic field provided by either peemanent magnets (or) Electromagnets. The ends of the coil are connected to two sliprings, a & b fixed on shaft. The brushes b_1 & b_2 (of carbon (or) copper) press against the slip rings. Their function is to collect the current induced in the coil and to convey it to the external load.



The rotating coil may be called the armature and the magnets as field magnets.

Working :- when coil is rotating in clockwise direction, the flux linking the coil changes continuosly and hence, an emf induced in the coil.

when coil is at position 1 ie., coil is vertical, the flux linking the coil is maximum, but the rate of changeof flux linkages is minimum. The reason is that, in this position the coil sides AB & CD don't cut the flux ie., they are more parallel to them. There fore emf induced in the coil is zero. This is the starting position.



As the coil continuous moving, the rate of chage of flux linkages (& hence emf in it) increase gradually till position 3 is reached where $\theta = 90°$, Here the coil plane is parallel to the lines of flux, the flux linked with the coil is minimum, but rate of change of flux linkages is maximum. Hence maximum emf is induced in the coil when in this position.

In the next Quarter revolution ie, from 90° to 180° the flux linked with the coil gradually increases, but the rate of change of flux linkages decreases. Therefore induced emf decreases gradually till position 5 of the coil. It is reduced to zero value.

So, in the first half revolution of the coil, no emf is induced in it, when in position 1, maximum emf induced when in position 3, & no emf induced when in position 5. The direction of induced emf can be determined by applying Fleming's Right hand rule which gives direction from A to B & C to D. Hence the direction of current from is ABMLCD ie, current through load R flows from M to L.

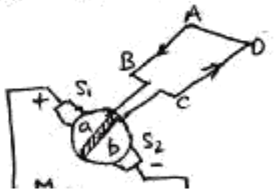
In the next half revolution, ie, from 180° to 360°, the variations in the magnitude of emf are similar to those in the first revolution. It value is maximum when in position 7 and minimum when in position 1. It is seen that direction of induced current is from D to C & B to A. Hence, the path of current flow is along DCLMBA & current through load R is from L to M.

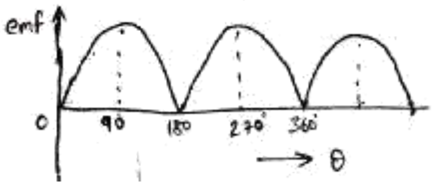
Therefore we can observe that the current which we obtain from such a simple generator reverses its direction after every half revolution. Such a current is called Alternating current.

For making the flow of current into unidirectional in the external circuit, the slip rings are replaced by split rings. These split rings made of conducting cylinder which cut into two halves or segments insulated from each other by a thin sheet of mica.

So in the first half revolution current flows along ABLMCD ie., brush a, which is in contact with segment S_1 and brush b in contact with segment S_2. Therefore brush a is positive & b is negative end of supply.

In the next half revolution, the direction of induced ~~emf~~ current in the coil is reversed But at the same time the positions S_1 & S_2 are also reversed that is brush a in contact with segment S_2 & Brush b with segment S_1. Hence the current in the load again flows ~~in the load~~ from L to M.





Production of Torque in DC Machine:-

The flow of Direct current in the field winding of a DC Machine creates a magnetic flux distribution called the field flux, which is stationary w.r.t the stator. Similarly the effect of the commutator in a dc machine is such that direct current flows through the brushes, the armature creates a magnetic flux distribution called as armature flux, which is fixed in space. The armature flux & field flux are perpendicular and their interaction creates the torque. The torque is the result of the tendency of these 2 flux distributions to align along same axis. If the machine is acting as a generator, this torque opposes the rotation produced by the driving torque of the prime mover. This phenomenon also conforms to lenz's law, as the torque opposes the very cause of its production, that is, the emf and current generated by rotation. If the dc machine is working as motor the electromagnetic torque is developed due to field flux and armature flux produced by the DC current fed to the armature from the external dc source, and the rotor armature starts to rotate in the same direction as the electromagnetic torque.

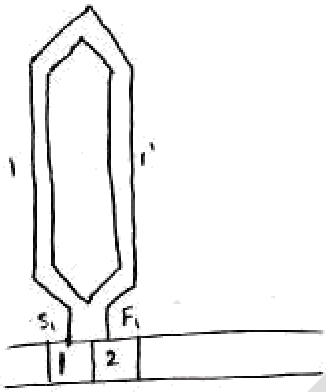
Armature windings:-

classified according to the commutator segment connections

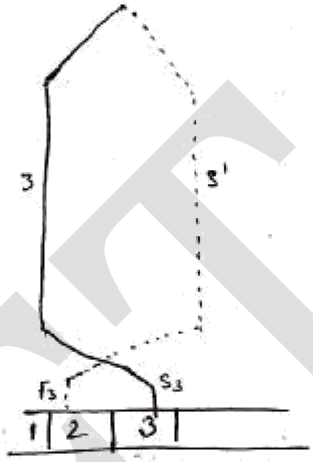
① Lap winding —— Progressive winding
Retrogressive lap winding

② Wave winding

① LAP:-



Progressive



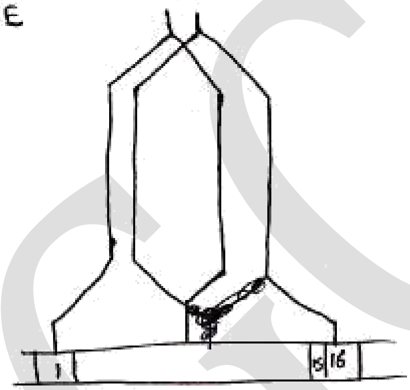
Retrogressive

No of parallel paths more

for a given size of cond
P time more ct in lap
emf induced is less/parallel path

LV & H Ct Equipment

② WAVE



Each parallel path distributed symmetrically over all the poles
⇒ flux uniform throughout & emf identical

HV & L Ct Equipment

Differences:-

Lap winding	Wave winding
① Number of parallel paths Equals to Number of poles	① Number of parallel paths are always two
② The Number of brush positions on the Commutator equals the numbers of poles	② A minimum of two brush positions are required irrespective of the no of poles.
③ The two ends of an armature coil are Connected to the two adjacent commutator segments	③ The two ends of an armature coil are Connected to the two commutator segments which are two pole pitches apart.
④ The winding forms a continuous closed ckt.	④ The winding forms a continuous closed ckt.
⑤ The lap wound generators are used for supplying Low Voltage & High current Loads.	⑤ The wave wound generators are used for supplying high voltage, Low current Loads

Expression for generated emf:-

Principle - Faradays laws of Electro magnetic Induction,

emf is induced in armature conductors when armature is rotated such as to cut the magnetic flux.

Let if ϕ — useful flux per pole in wb

$2p$ — Total number of poles

Z — Total number of armature conductors

N — Speed of the armature in revolutions per min (rpm)

E — Total emf generated

Then Average emf generated per conductor

$$e = \frac{d(N\phi)}{dt} = \frac{d\phi}{dt} \text{ Volts, if } N = 1 \text{ rpm}$$

When armature completes one revolution, each cond on the armature cuts a flux of $2p\phi$. No of revolutions made by armature/sec $= \frac{N}{60}$

∴ flux cut by each cond in one second is

⇒ flux cut/rev × Number of revol/sec

⇒ $2p\phi \times \frac{N}{60}$

Average emf generated

$$e = \frac{d\phi}{dt} = \frac{2p\phi N}{60} \text{ volts.}$$

If $2a$ is no of parallel paths, then no of cond in series/parallel path is $\frac{Z}{2a}$

Then average emf across brushes $= 2p\phi\frac{N}{60} \times \frac{Z}{2a}$

emf generated in dc machine, $e = \frac{\phi Z N}{60}\frac{P}{A}$

$A = P$ in Lap

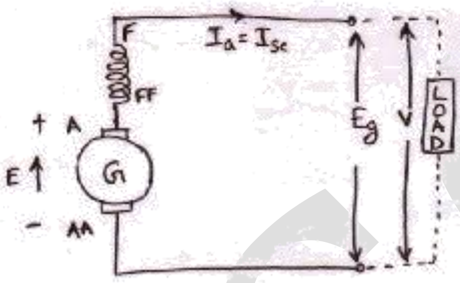
$A = 2$ in wave

Classification of D.C. Machine

- D.C Machine
  - D.C. Generator
  - D.C. Motor
    - Series
    - Shunt
    - Compound
      - Cumulative compound
      - Differential compound
        - Long Shunt
        - short shunt

D.C. Generator :-

Series Generator :-

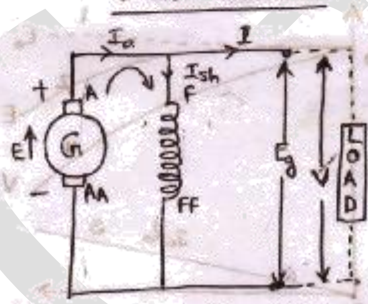


$V = E - I_a(R_a + R_{se})$

$P = E \cdot I_a$

$I_a = I_{se}$

Shunt Generator :-



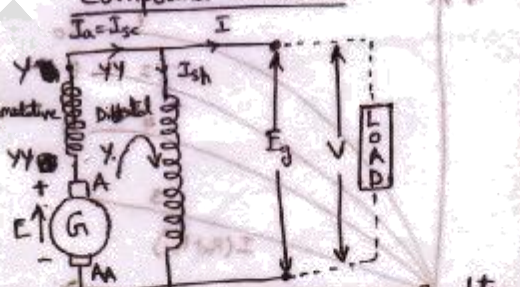
$V = E - I_a R_a$

$P = E \cdot I_a$

$I_L = I_a - I_{sh}$

$I_{sh} = \frac{V_{sh}}{R_{sh}}$

Compound Generator :-



Connection type Y—YY—A—AA—Cumulative
YY—Y—A—AA—Differential

Short shunt :-

$V = E - I_a R_a - I_L R_{se}$

$I_a = I_{sh} + I_L$

$I_{sh} = \frac{E - I_a R_a}{R_{sh}}$

Long shunt :-

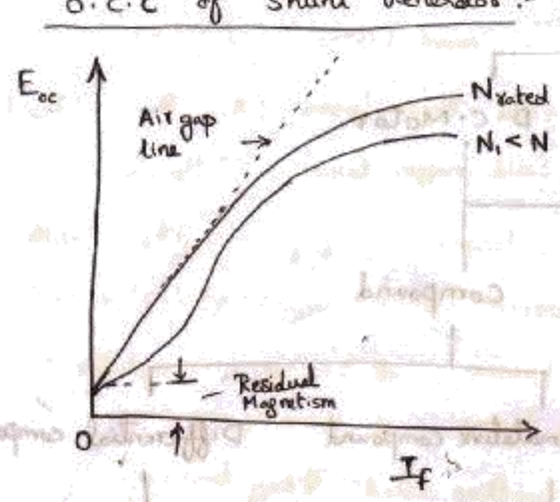
$I_a = I_{sh} + I_L$

$V = E - I_a R_a - I_a R_{se}$

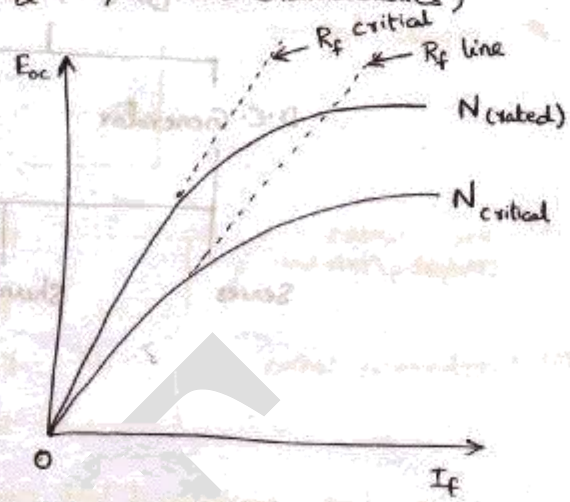
$I_{sh} = \frac{V_{sh}}{R_{sh}}$

Characteristics of DC Generator :-

O.C.C of Shunt Generator :- (Magnetization / No-Load characteristics)



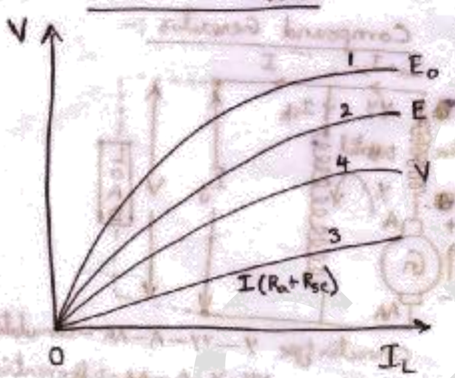
Seperatly Excited Machine



Self-Excited Machine

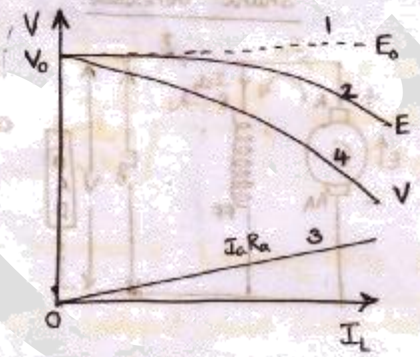
Load characteristics :-

Series Generator :-



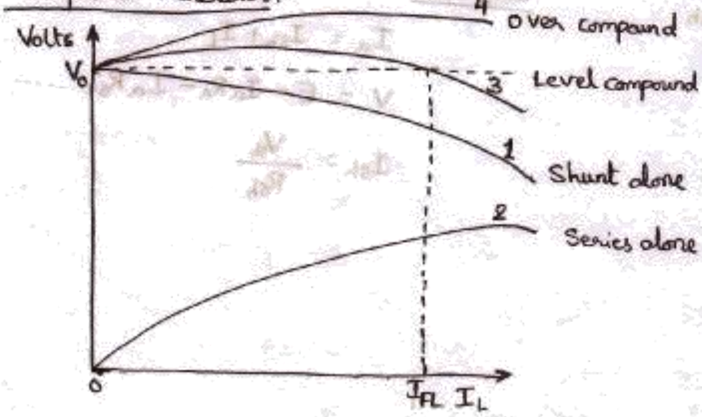
1) OCC characteristic (E_0)
2) Induced emf (E) at No load
3) Internal voltage drop ($I_a(R_a+R_{sc})$)
4) External characteristics ($E - I_a(R_a+R_{sc})$)

Shunt Generator :-



① No-Load (Internal characteristics)
② Drooping characteristics $I_L\uparrow - I_a\uparrow \rightarrow \phi/\text{pole}\downarrow$
③ Internal Voltage drop (I_aR_a)
④ External characteristics ($E - I_aR_a$)

Compound Generators :-



① Shunt field Alone (External characteristics) drooping
② Series field Alone (rising characteristics)
③ Level compound ($V_{TNL} = V_{TFL}$)
④ Over Compound (strong series field, $V_T\uparrow - I_L\uparrow$)

Losses in a DC Generator :-

① Copper losses
- Armature copper losses ($I_a^2 R_a$) W
- Shunt field copper losses ($I_{sh}^2 R_{sh}$ / $V I_{sh}$) W
- Series field copper losses $I_{se}^2 R_{se}$ W

② Iron losses (Magnetic / core loss)
- Hysterisis losses $W_h = \eta B_m^{1.6} \cdot f \cdot V$ Watts
- Eddy current losses $W_e = B_m^2 \cdot f^2 \cdot t^2 \cdot V$ Watts

③ Mechanical losses
- Friction losses at bearings & Commutator
- Windage losses of rotating armature

Iron loss + Mechanical loss combined together called stray losses.
For shunt & compound generators, field copper losses are constant. } Constant losses

Total losses = Armature cu loss + Constant losses

Power stages :-

mechanical, $\eta_m = \dfrac{\text{Total watts generated in armature } (E I_a)}{\text{Mechanical power supplied } (I/P)}$

Electrical $\eta_e = \dfrac{O/P}{I/P} = \dfrac{VI}{E I_a}$

overall (or) commercial $\eta_c = \dfrac{VI}{\text{Mech } I/P}$

$\eta = \eta_m \cdot \eta_e$ 95%

Problems :- (DC Generator)

a) A Six pole, Wave connected armature has 200 conductors and runs at 1500 rpm. The emf generated in the open circuit is 600V. Find the useful flux per pole.

b) An eight-pole, Lap-Connected armature has 800 conductors, a flux of 0.05 wb/pole, and a speed of 500 rpm. Calculate the emf generated in the open circuit.

c) If the armature in (b) is wave connected, at what speed must it be driven to generate 400V?

d) A four pole generator has a flux of 0.05 wb/pole and a Lap connected armature with 600 conductors. Find the emf generated in the open circuit at 800 rpm.

Sol :-

We Know $E = \frac{\phi Z N P}{60\ A}$

a) $\phi = \frac{E \times 60 A}{Z N P} = \frac{600 \times 60 \times 2}{200 \times 1500 \times 6} = 0.04$ wb

b) $E = \frac{0.05 \times 800 \times 500 \times 8}{60 \times 8} = 333.33V$

c) $N = \frac{E \times 60 A}{\phi Z P} = \frac{400 \times 60 \times 2}{0.05 \times 800 \times 8} = 150$ rpm

d) $E = \frac{0.05 \times 600 \times 800 \times 4}{60 \times 4} = 400V$

2) A 4 pole Shunt generator with Lap connected armature having field and Armature Resistances of 100Ω and 0.05Ω respectively, supplies 100 Lamps each rating 40W, 200V. Calculte total armature current, the armature current per path, & emf generated. Assume constant 1V/brush drop.

Sol :-

Given, $P = 4$; $A = 4$; $R_a = 0.05\Omega$; $R_{sc} = 100\Omega$; brush drop = 2V

Total load current $I_L = \frac{100 \times 40}{200} = 20A$

$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{100} = 2A$

$\therefore I_a = I_L + I_{sh} = 20 + 2 = 22A$

current / parallel path $= \frac{I_a}{A} = \frac{22}{4} = 5.5A$

Generated emf

$E = V + I_a R_a + V_{brushdrop} = 200 + 22 \times 0.05 + 2 = 203.1V$

③ A series generator is delivering 5 KW to heater load at 200 V when operating at 1000 rpm. If the speed is raised to 1200 rpm & the power delivered to the same heater load increases to 6 KW, determine the armature current and the voltage across the load. The total armature and series field resistances of the generator is 0.5 Ω.

Sol:-

Given,

$$\text{Load} = 5\,\text{KW} = 5000\,\text{W} \;;\; V = 200\,\text{V} \;;\; R_a + R_{se} = 0.5\,\Omega$$

$$N_1 = 1000\,\text{rpm} \;;\; N_2 = 1200\,\text{rpm}$$

$$I_{a_1} = \frac{5000}{200} = 25\,\text{A}$$

$$V_{drop} = 25 \times 0.5 = 12.5\,\text{V}$$

Generated emf :-

$$E_1 = V + I_{a_1} R_a = 200 + 12.5 = 212.5\,\text{V}$$

Load, $I_{a_1}^2 R_L = (25)^2 \times R_L = 5000\,\text{W}$

$$R_L = \frac{5000}{625} = 8\,\Omega$$

When power delivered $= 6000\,\text{W} = I_{a_2}^2 R_L$

$$\Rightarrow I_{a_2}^2 = \frac{6000}{8} \Rightarrow I_{a_2} = \sqrt{\frac{6000}{8}} = 27.386\,\text{A}$$

We know

$$E_1 \propto \phi_1 N_1$$

for series generator, $\phi_1 \propto I_{a_1}$

$$\Rightarrow E_1 \propto I_{a_1} N_1 \;;\; ||^{ly}\; E_2 \propto I_{a_2} N_2$$

$$\therefore \frac{E_2}{E_1} = \frac{I_{a_2} N_2}{I_{a_1} N_1} \Rightarrow E_2 = E_1 \times \frac{I_{a_2} N_2}{I_{a_1} N_1} = \frac{27.386 \times 1200}{25 \times 1000} \times 212.5 = 279.34\,\text{Volts}$$

4) A 4-Pole, 400V, Shunt generator has 720 Wave Connected Conductors in its armature. The Full load current is 80A and the flux/pole is 0.03 wb. $R_a = 0.1\,\Omega$ & Contact drop is 1V/brush. Calculate the full load speed of the motor.

Sol: Given, $P = 4$; $A = 2$; $Z = 720$; $I_L = 80\,\text{A}$; $\phi = 0.03\,\text{wb}$; Brush drop $= 2\,\text{V}$; $V = 400\,\text{V}$

Back emf, $E = V - I_a R_a = 400 - 80 \times 0.1 - 2 = 390\,\text{V}$

$$E = \frac{\phi Z N P}{60 A} = \frac{0.03 \times 720 \times N \times 4}{60 \times 2} \Rightarrow 390 = 0.72 N$$

$$\Rightarrow \therefore N = \frac{390}{0.72} = 541.67\,\text{rpm}$$

6. D.C. Motor:

If the armature terminals of a dc Machine are connected to a dc Source, It begins to rotate and operate like a motor. Converting electrical Energy into mechanical Energy. Construction wise, a dc motor is similar to a DC generator. Since the former has to operate in stringent Environmental conditions, It has to be protected against moisture, fire hazards, chemical gases and mechanical damages. Therefore, the frame of a dc motor is either fully (or) partially closed to provide sufficient protection and is made flameproff.

Principle of operation:-

It operation is based on the principle that, "whenever a current carrying conductor placed in a magnetic field, it will experiences a force whose direction is given by flemings left hand rule.

① Pole Field ② Conductor Field ③ ④ Resultant field & force of action

On the upperside of the conductor, in fig ③ the magnetic lines of force and field exists around the conductor are additive, while on the lower side these are substractive. This explains the resultant field is strengthened above and weakened below the conductor.

From the fig ③ it shows that the conductor has a force on it which tends to move it downwards. This displays the force acts in the direction of the weaken field. When the current in the conductor is reversed, the direction of force is also reversed as shown in figure ④

The Force (F) developed in the conductor is given by the relation

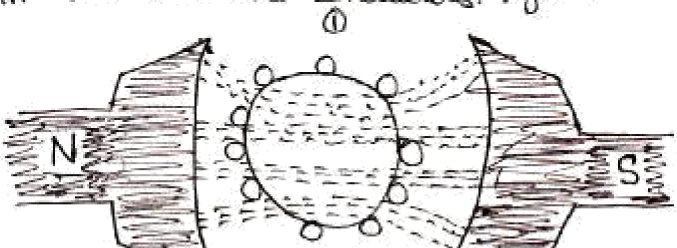
$F = BIl$ Newtons

where B – Flux density in wb/m^2

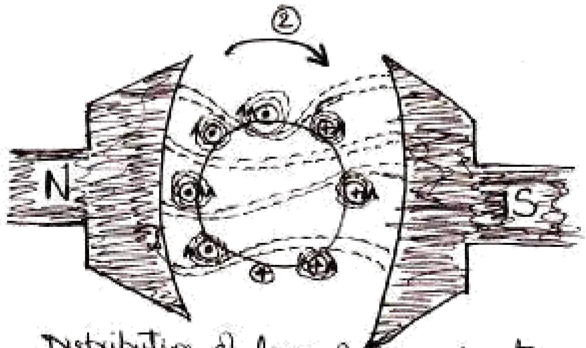
I – Current in amperes

l – length of the conductor in meters

Now, consider the magnetic field of a dc motor in which there is no current in the armature conductors. figure ①



Distribution of lines of force due to Magnetic Field alone.



Distribution of lines of force due to armature and Magnetic Field, on load.

When, the armature conductors carry current. All the conductors under North Pole are assumed to carry current upwards (dots) and those under South Pole to carry current downwards (crosses). Each of its conductor carry a magnetic field which, when superimposed on the mainfield. Therefore, main magnetic field is distored as shown in second figure.

Each conductor experiences a force F, which tends to rotate the armature in clockwise direction. All these forces add together to produce a driving torque which sets the armature rotating.

Types of DC Motors:-

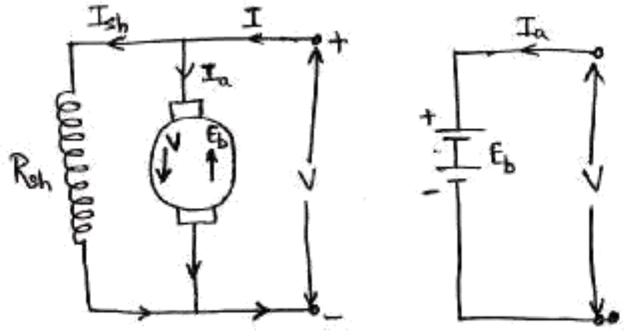
① Series wound Motor
② Shunt wound Motor
③ Compound wound Motor.

Back (or) Counter EMF :-

When the Motor armature rotates, the armature conductors cut the flux, and as a result an emf is induced in them.

The direction of this emf induced is opposite to that of the applied voltage, V. So it is called as back (or) counter emf, denoted by E_b.

The magnitude of this back emf may be calculated from the same emf Equations used for generator.

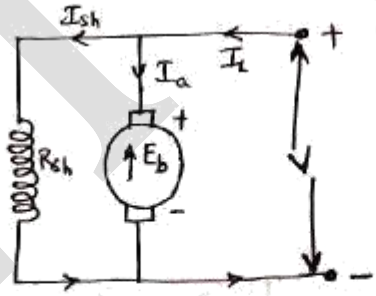


$$E_b = \frac{\phi Z N}{60}\left(\frac{P}{A}\right) \text{ Volts.}$$

where E_b – back emf
ϕ – flux/pole in wb
N – Speed in revolutions, rpm
P – No of poles
A – No of parallel paths
Z – Total number of conductors.

Voltage Equation of D.C. Motor:-

The voltage applied across the armature to (i) Overcome the back emf, E_b
(ii) Supply the armature ohmic drop, $I_a R_a$



$$\therefore \quad V = E_b + I_a R_a$$

This is known as Voltage Equation of Motor

where,
- V – Applied voltage
- I_a – Armature current
- I_{sh} – Shunt field current
- E_b – back emf
- R_a – Armature Resistance
- R_{sh} – shunt Field Resistance.

Power Relationship in a Motor:

The Electrical power supplied to the armature is VI_a (armature I/p) &

$$VI_a = (E_b + I_a R_a) \cdot I_a$$

$$= E_b I_a + I_a^2 R_a$$

$I_a^2 R_a$ ← Electrical power wasted in armature (Copper loss in armature)

$E_b I_a$ ← Electrical Equivalent of Mech power in armature

∴ out of the armature I/p, a small portion is wasted as $I_a^2 R_a$ and the remainder is available as Mech power in armature $\therefore P_{mech} = VI_a - I_a^2 R_a$

Speed of d.c. Motor:-

When a motor is running, the back emf is always less than the applied voltage.

ie., $E_b = V - I_a R_a$

But $E_b = \frac{\phi Z N}{60}\left(\frac{P}{A}\right)$ Volts

As Z, P & A are constants,

$$E_b \propto \phi \cdot N$$

(or)

$$N \propto \frac{E_b}{\phi} \qquad \text{(Flux control)}$$

Therefore, the speed of a d.c. Motor is directly proportional to E_b and inversely proportional to flux/pole, ϕ.

$$E_b = V - I_a R_a$$

$$\Rightarrow N \propto \frac{(V - I_a R_a)}{\phi} \qquad \text{(Armature control)}$$

If initial values of speed, flux per pole & back EMF are, N_1, ϕ_1 & E_{b_1} & final values are N_2, ϕ_2 & E_{b_2}

Then $N_1 \propto \frac{E_{b_1}}{\phi_1}$; $N_2 \propto \frac{E_{b_2}}{\phi_2}$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$$

For series Motor, $\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{I_{a_1}}{I_{a_2}}$ $\left[\because \phi_1 \propto I_{a_1} \text{ \& } \phi_2 \propto I_{a_2}\right]$

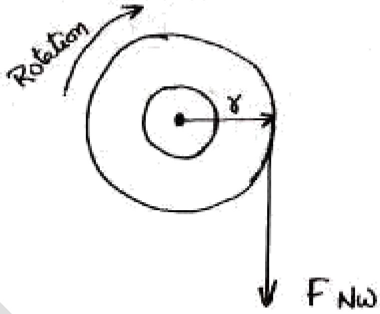
For shunt Motor, $\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}}$ $\left[\because \phi_1 = \phi_2 = \text{Constant}\right]$

Torque in a DC Motor:-

General defination:- "Torque means the turning (or) twisting moment of a force about an axis"

Torque is measured by the product of force and the radius at which this force acts.

Consider a wheel of radius (r) meters acted upon by a circumferential force F Newtons as shown. Let this force cause the wheel to rotate at N r.p.s.



Torque, $T = F \times r$ Newton-meters

Work done per revolution $=$ Force $\times$ distance moved

$= F \times 2\pi r$ Joules

Work done per second $= F \times 2\pi r \times N$

$= (F \times r) \times 2\pi N$

$= T \times 2\pi N$ Joules/sec $2\pi N = \omega$ - angular velocity in radians/second

Power(P) $= T \cdot \frac{2\pi N}{60} = 0.105\, N \cdot T$ watts

Armature Torque:-

Let T_a be the torque developed in Nw-m by the motor armature running at N rps.

Power developed $=$ Work done per second

$= T_a \times 2\pi N$ watts — ①

Electrical power converted into mechanical power in the armature $= E_b I_a$ watts ②

Comparing ① & ②

$$T_a \times 2\pi N = E_b \cdot I_a$$

$$T_a = \frac{E_b I_a}{2\pi N} = \frac{\phi Z N P \cdot I_a}{2\pi N \cdot A}$$

$$= \frac{1}{2\pi} \cdot \phi \cdot Z \cdot I_a \left(\frac{P}{A}\right) \text{ Nw-m}$$

$$\therefore T_a = 0.159\, \phi Z I_a \left(\frac{P}{A}\right) \text{ N-m}$$

Shaft Torque:- The Torque which is available at the motor shaft for doing useful work is known as Shaft Torque & denoted by "T_{sh}"

$T_{sh} = T_a - T_f$

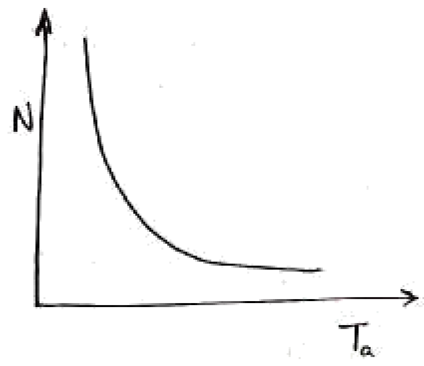
$\because$ Z, P, A are constant for a perticular machine, $\therefore T_a \propto \phi I_a$

Series motor $\longrightarrow T_a \propto \phi I_a \Rightarrow T_a \propto I_a^2$

Shunt motor $\longrightarrow T_a \propto I_a$

③ N/T characteristics:- With the help of T/I_a curve & N−I_a curve we can draw the N/T characteristic curve.

As the torque increases, speed decreases.



DC Shunt Motor :-

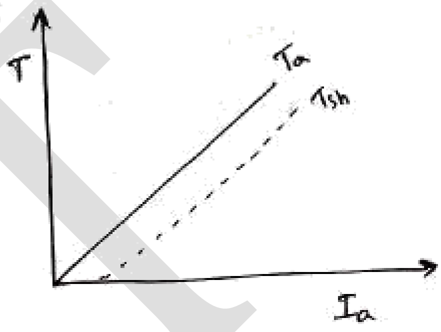
① T/I_a Characteristics :-

$$T_a \propto \phi I_a$$

∵ ϕ is constant in shunt motor, $T_a \propto I_a$

∴ curve is a st line passing through the origin.

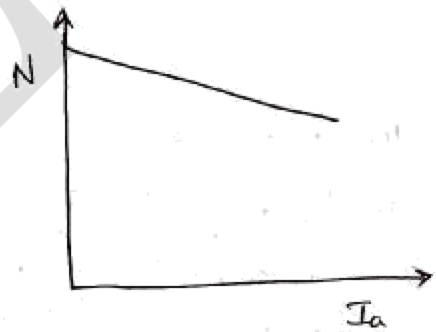
Shaft torque is less than armature torque.



② N/I_a characteristics :-

$$N \propto \frac{E_b}{\phi}$$

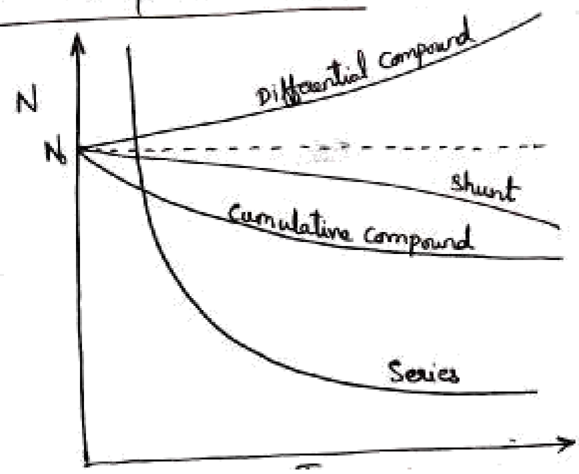
$$\Rightarrow N \propto E_b \quad [\because \phi \text{ is constant}]$$



③ N/T characteristics :-

From T/I_a & N/I_a characteristics →

DC Compound Motor :-



Cumulative compound :-

Series field aids shunt field, ϕ/pole ↑ as I_a ↑

∴ Curve between shunt & series (where $\phi \propto I_a$)

Differential compound :-

With increase of Load, Armature drop ($-I_aR_a$) ↑ which tries to decrease speed, but at the same time demagnetizing effect of I_a & Series field tries to decrease Excitation turns, ⇒ Net increase in speed.

Necessity of Starter:-

The current drawn by the motor armature is given by $I_a = \frac{V - E_b}{R_a}$

When motor is directly connected to the supply, there is no back emf in the beginning to oppose the supply voltage. The result is that heavy current will flow through the armature conductors and will damage it since resistance of motor armature is very low.

Therefore, the starting armature current is $I_a = \frac{V}{R_a}$.

for Eg. consider a 5HP, 220V motor having armature resistance of 0.5Ω

$$\text{full load current } I_f = \frac{5 \times 746}{220} = 16.95\text{ A}$$

$$\text{starting current, } I_s' = \frac{220}{0.5} = 440\text{ A}$$

$$\frac{I_s}{I_f} = \frac{440}{16.95} \simeq 26$$

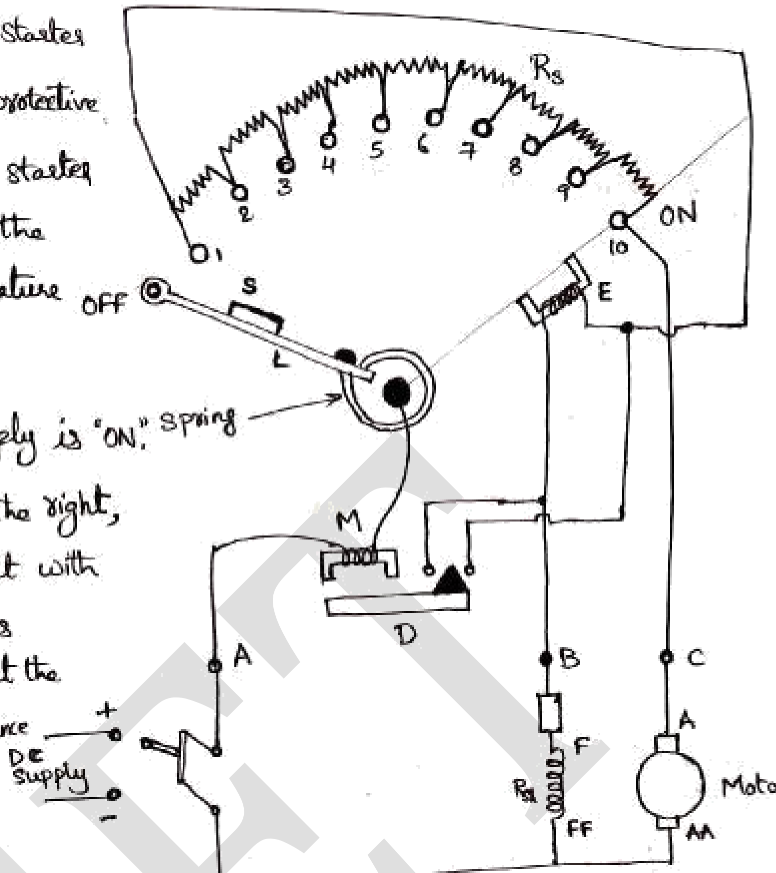
$$I_s = 26\, I_f$$ [That means, starting current is 26 time F.L cur]

This high current will cause high sparking at the commutator. It effects would be to damage the segments & burn the brushes.

So in order to avoid excessive current at starting, a variable resistance is added in series with the armature for the duration of starting period only. It limits the starting current to safe value. The starting resistance is gradually cut in steps as the motor gains speed and develops back emf and ultimatly when motor attainging its normal speed, the starting resistance is totally cut out from the armature circuit.

Three point Starter :-

The Figure shows the 3-point starter for a dc Shunt motor with protective devices. The 3 terminals of the starter A, B & C are connected to the Positive Line, shunt field & Armature terminals respectively.



To start the motor d.c supply is "ON." The starter arm is moved to the right, as soon as it comes in contact with stud no. 1. The field circuit is connected across the line and at the same time the entire starting Resistance is inserted in the armature circuit. As the handle is gradually moved over to final stud, the starting resistance is cutout of armature circuit in steps. when the handle comes in contact with final stud, entire R_s is cutout of armature ckt.

No-Volt Release coil (NVRC, E) :- It consists of Electromagnet connected in series with field winding and therefore carries field current. There is a soft iron piece "S" attached to the arm which is in full "ON" (or) running position is attracted and held by the "No-Volt Release".

Now When Supply fails, (or) gets disconnected the electromagnet demagnetises and so releases the starting arm, which goes back to "OFF" position due to spring attached to it and gets disconnected from supply mains.

Over-Load Release coil (OLRC, M) :- It is connected in series with the motor and carries the full load current. If the motor becomes overloaded, beyond certain value, then D is lifted and short circuits the NVRC. The coil demagnetises and the starter arm is released to OFF position with the action of spring attached to it and the motor is automatically disconnected from the supply.

Efficiency (η) when running as a Motor:-

Load current at which η is required $= I$

Armature current (I_a) $= I - I_{sh}$

Motor Input $= V \cdot I$

Armature copper losses $= I_a^2 R_a = (I - I_{sh})^2 \cdot R_a$

Total losses $= P_{constant} + (I - I_{sh})^2 \cdot R_a$

$$\eta_{motor} = \frac{Output}{Input} = \frac{Input - Total\ losses}{Input}$$

$$= \frac{\{VI - [P_{constant} + (I - I_{sh})^2 R_a]\}}{VI}$$

Efficiency (η) when running as a generator:-

Load current at which η is required $= I$

Armature current (I_a) $= I + I_{sh}$

Generator Input $= V \cdot I$

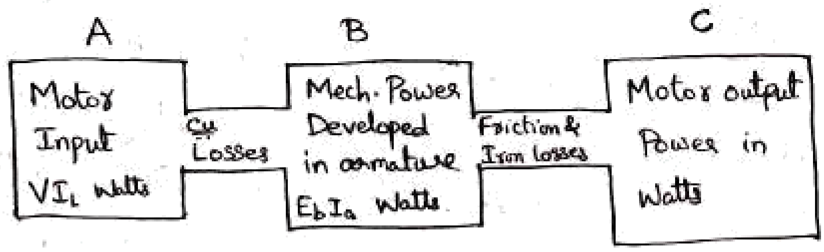
Armature copper losses $= I_a^2 R_a = (I + I_{sh})^2 \cdot R_a$

Total losses $= P_{constant} + (I + I_{sh})^2 \cdot R_a$

$$\eta_{Generator} = \frac{Output}{Input} = \frac{output}{output + total\ losses}$$

$$= \frac{VI}{[VI + \{P_{constant} + (I + I_{sh})^2 \cdot R_a\}]}$$

Power stages



A – B : Copper losses

B – C : Iron & Friction losses

Overall $\eta_c = \frac{C}{A}$

Electrical $\eta_e = \frac{B}{A}$

Mechanical $\eta_m = \frac{C}{B}$

③ A d c Motor having a terminal voltage of 230 V and the armature current of 50 A has a back emf of 225V, calculate

(i) Armature resistance

(ii) Power developed in motor in watts

(iii) Power developed in motor in Horse power

Sol:-

$V = 230V$; $I_a = 50A$; $E_b = 225V$

(i) $R_a = \frac{V - E_b}{I_a} = \frac{5}{50} = 0.1\Omega$

(ii) Power in watts $= E_b \times I_a = 225 \times 50 = 11.25 KW$

(iii) Power in Horse Power $= \frac{E_b I_a}{746} = \frac{11,250}{746} = 15$ H.P.

④ Determine the value of torque in N-m established by armature of 4-Pole motor having 750 conductors, two parallel paths, 20 mwb/pole when $I_a = 60A$.

$P = 4$; $Z = 750$; $A = 2$; $\phi = 20 \times 10^{-3}$ wb ; $I_a = 60A$

$T = ?$

We Know $T = 0.159 \, \phi Z I_a \left(\frac{P}{A}\right)$ N-m

$= 0.159 \times 20 \times 10^{-3} \times 750 \times 60 \times \left(\frac{4}{2}\right)$

$\therefore T = 286.2$ N-m

⑤ A 500V dc motor takes an armature current of 60A when its speed is 800rpm. If the armature resistance is 0.2 Ω. Calculate the torque developed.

$V = 500V$; $I_a = 60A$; $N = 800$ rpm ; $R_a = 0.2\Omega$

$E_b = V - I_a R_a = 500 - 60 \times 0.2 = 488V$

Torque developed $T = \frac{E_b I_a}{2\pi N/60} = \frac{488 \times 60}{2 \times \pi \times 800/60}$

$= 349.5$ N-m.

Problems:- (DC Motors)

① A 220V DC Shunt motor has an armature resistance of 0.5Ω. If full load armature current is 25A and the no-load armature current is 3A. Find the change in back emf from No Load to Full Load.

Sol:- Rated voltage of the motor, V = 220V

① When motor is on Full Load,

Full load armature current $I_{a_1} = 25A$

Armature resistance, $R_a = 0.5Ω$

∴ Back emf $= V - I_a R_a = 220 - 25 \times 0.5$

$= 207.5V$

② When motor is on No-load;

No load armature current, $I_{a_2} = 3A$

Back emf, $E_{b_2} = V - I_{a_2} R_a = 220 - 3 \times 0.5$

$= 218.5V$

Hence, change in back EMF from NL to FL is $E_{b_2} - E_{b_1} = 218.5 - 207.5$

$= 11$ Volts.

② A 230V DC Shunt motor takes 32A at fullload. Find the backemf on Full load if the resistance of motor armature and shunt field winding are 0.2Ω & 115Ω respectively.

Sol:- Supply Voltage, V = 230V

FL current, $I = 32A$

$R_a = 0.2Ω$ & $R_{sh} = 115Ω$

$E_b = ?$

Shunt field current $I_{sh} = \frac{V}{R_{sh}} = \frac{230}{115} = 2A$

Armature current, $I_a = I - I_{sh} = 32 - 2 = 30A$

Back EMF, $E_b = V - I_a R_a$

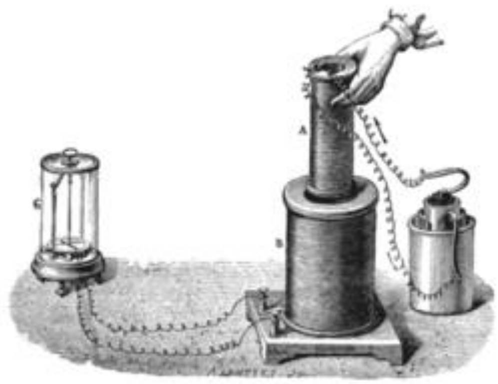
$= 230 - 30 \times 0.2$

$= 224V$ on F.L.

Unit 5 TRANSFORMERS

A **transformer** is a device that transfers electrical energy from one circuit to another through inductively coupled conductors—the transformer's coils. A varying current in the first or primary winding creates a varying magnetic flux in the transformer's core and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force (EMF), or "voltage", in the secondary winding. This effect is called inductive coupling.

Discovery



Faraday's experiment with induction between coils of wire

The phenomenon of electromagnetic induction was discovered independently by Michael Faraday and Joseph Henry in 1831. However, Faraday was the first to publish the results of his experiments and thus receive credit for the discovery. The relationship between electromotive force (EMF) or "voltage" and magnetic flux was formalized in an equation now referred to as "Faraday's law of induction":

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right|$$

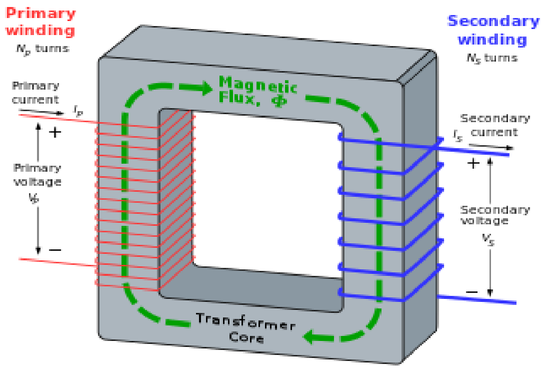
where $|\mathcal{E}|$ is the magnitude of the EMF in volts and Φ_B is the magnetic flux through the circuit in webers.

Faraday performed the first experiments on induction between coils of wire, including winding a pair of coils around an iron ring, thus creating the first toroidal closed-core transformer.

WORKING PRINCIPLE OF TRANSFORMER:

Introduction

The main advantage of alternating currents over direct current is that, the alternating currents can be easily transferable from low voltage to high voltage or high voltage to low. Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called transformer. The transformer works on the principle of mutual induction. It transfer an electric energy from one circuit to other when there is no electrical connection between the tow circuits. Thus we can define transformer as below :



Key point : The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.

The use of transformers in transmission system is shown in the Fig 1.1.

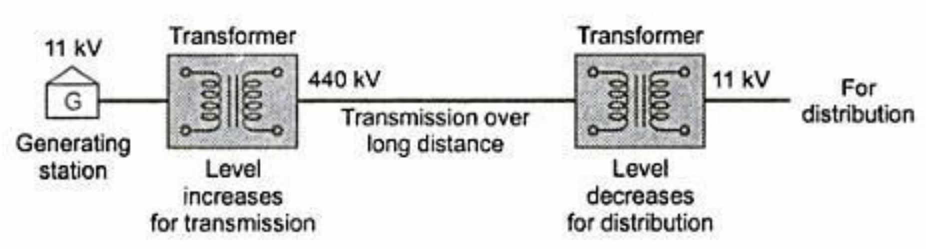


Fig. 1.1 Use of transformer in transmission system

PRINCIPLE OF WORKING

The principle of mutual induction states that when tow coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f can drive a current, when a closed path is provided to it. The transformer works on the same principle. In its elementary form, it consists of tow inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in the Fig 1.2.

One of the two coils is connected to source of alternating voltage. This coil in which electrical energy is fed with the help of source called primary winding (P). The other winding is connected to load. The electrical energy transformed to this winding is drawn out to the load.

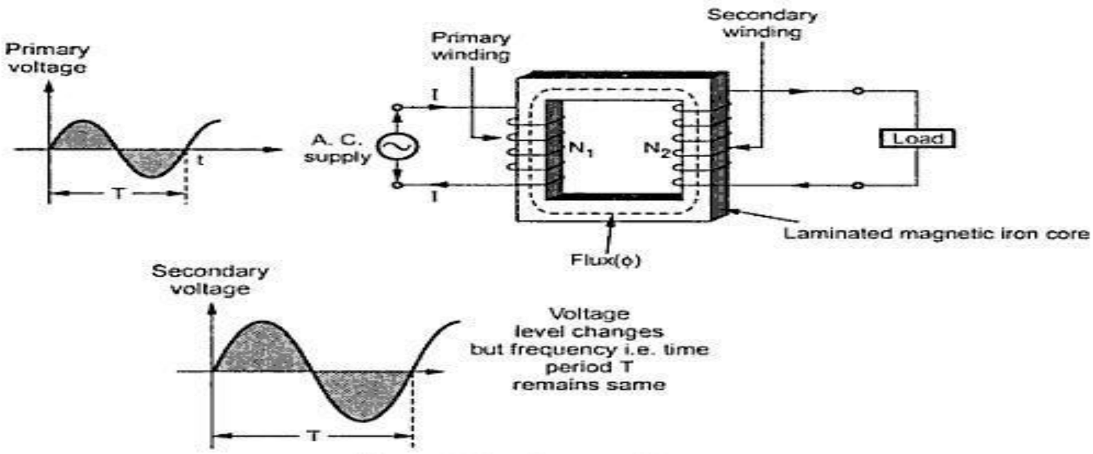


Fig.1.2 Basic transformer

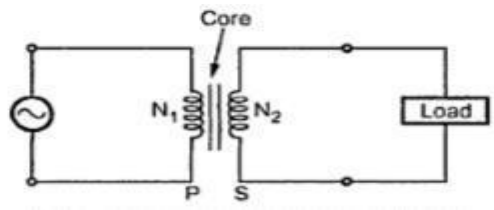


Fig 1.3 Symbolic representation

This winding is called secondary winding (S). The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns. Symbolically the transformer is indicated as shown in the Fig 1.3.

When primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (Φ)which completes its path through common magnetic core as shown dotted in the Fig 1.2. Thus an alternating, flux links with the secondary winding. As the flux is alternating, according to Faraday's law of an electromagnetic induction, mutually induced e.m.f. gets developed in the secondary winding. If now load is connected to the secondary winding, this e.m.f. drives a current through it.

Thus through there is no electrical contact between the two windings, an electrical energy gets transferred from primary to the secondary.

Key point : The frequency of the mutual induced e.m.f. is same as that of the alternating source which is supplying energy to the primary winding.

Can D.C. Supply be used for Transformer?

The d.c. supply can not be used for the transformers.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If d.c. supply is given, the current will not change due to constant supply and transformer will not work.

Practically winding resistance is very small. For d.c., the inductive reactance X_L is zero as d.c. has no frequency. So total impedance of winding is very low for d.c. Thus winding will draw very high current if d.c. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.

There can be saturation of the core due to which transformer draws very large current from the supply when connected to d.c.

Thus d.c. supply should not be connected to the transformers.

CONSTRUCTION OF TRANSFORMER:

There are two basic parts of a transformer i) Magnetic Core ii) Winding or Coils.

The core of the transformer is either square or rectangular in size. It is further divided into tow parts. The vertical position on which coils are wound is called limb while the top and bottom horizontal portion is called yoke of the core. These parts are shown in the Fig.1(a).

Core is made up of lamination. Because of laminated type of construction, eddy current losses get minimised. Generally high grade silicon steel laminations (0.3 to 0.5 mm thick) are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the air gap at joints. For this generally 'L' shaped or 'I' shaped laminations are used which are shown in the Fig 1(b).

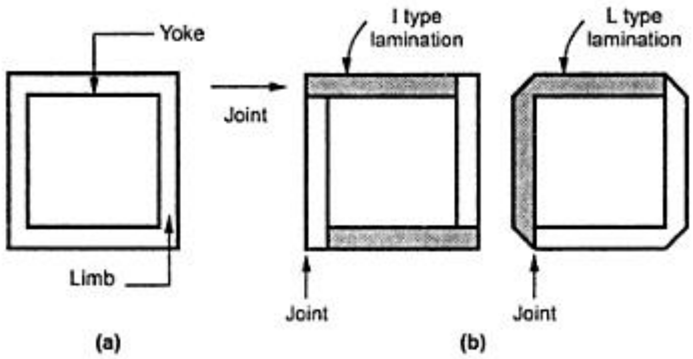


Fig. 1 Construction of transformer

The cross-section of the limb depends on the type of coil to be used either circular or rectangular. The different cross-section of limbs, practically used are shown in the Fig. 2.

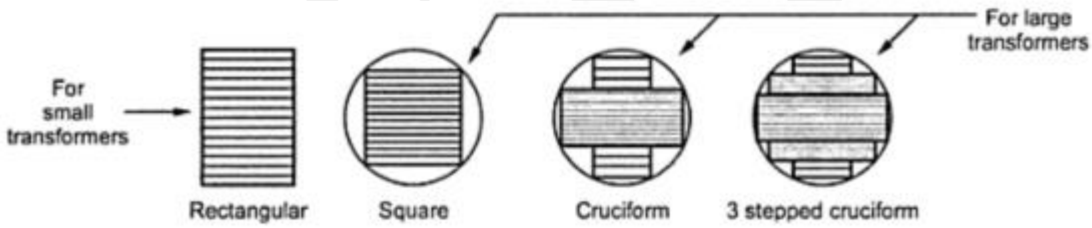


Fig. 2 Different cross-sections

Types of Windings

The coils used are wound on the limbs and are insulated from each other. In the basic transformer shown in the Fig 1.2 the two windings wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to this leakage flux increases which effects the transformer performance badly. Similarly it is necessary that the windings should be very closes to each other to have high mutual inductance. To achieve this, the two windings are split into number of coils and are wound adjacent to each other on the same limb. A very common arrangement is cylindrical coils as shown in the Fig. 3.

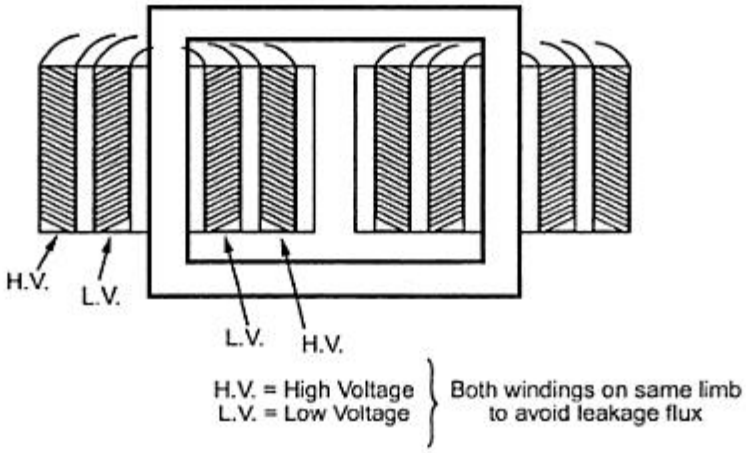


Fig. 3 Cylindrical concentric coils

Such cylindrical coils are used in the core type transformer. Theses coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

The other type of coils which is very commonly used for the shell type of transformer is sandwiching coils. Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portion reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The sandwich coil is shown in the Fig. 4. The top and bottom coils are low voltage coils. All the portion are insulated from each other by paper.

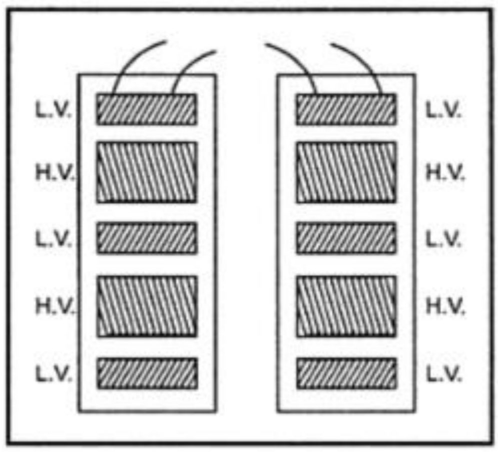


Fig. 4 Sandwich coils

The various types of depending on the construction of core used for the single phase transformers are,

1. Core type 2. shell type and 3. Berry type

1. Core Type Transformer

It has a single magnetic circuit. The core rectangular having two limbs. The winding encircles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layers insulated from each other by paper or mica. Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil. Core is made up of large number of thin laminations.

As The windings are uniformly distributed over the two limbs, the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

The Fig. 1(a) shows the schematic representation of the core type transformer while the Fig 1(b) shows the view of actual construction of the core type transformer.

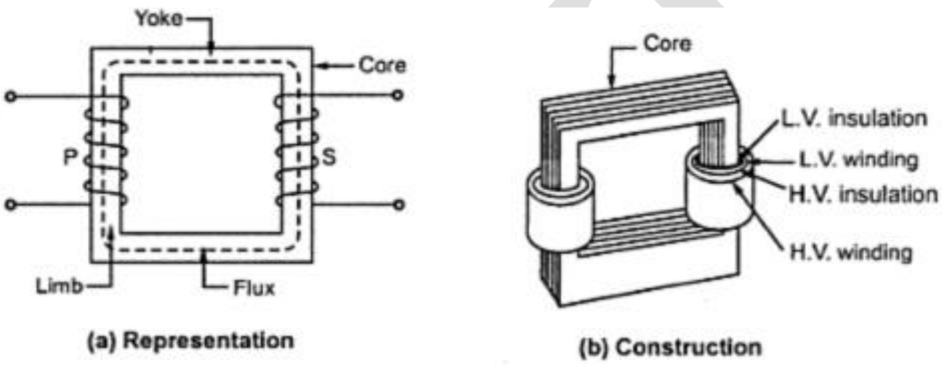


Fig. 1 Core type transformer

2. Shell Type Transformer

It has a double magnetic circuit. The core has three limbs. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils. As mentioned earlier, each high voltage coil is in between tow low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called over lapped or imbricated joint. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laimnations are required to be removed.

The Fig. 2(a) shows the schematic representation while the Fig. 2(b) shows the outaway view of the construction of the shell type transformer.

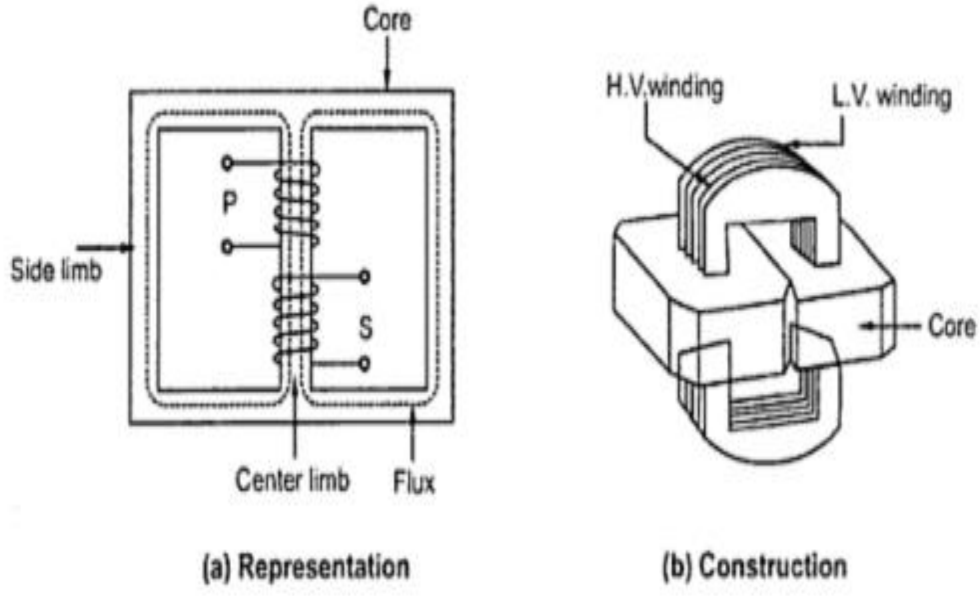


Fig 2 Shell type transformer

<u>3. Berry Type Transformer</u>

This has distributed magnetic circuit. The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

Diagramatically it can be shown as in the Fug. 3.

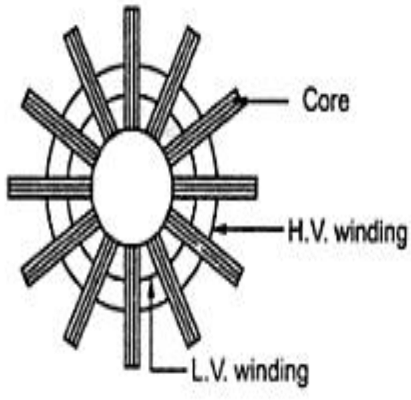


Fig. 3 Berry type transformer

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate cut, formed and welded into the rigid structures. All the joints are painted with a solution of light blue chalk which turns dark in the presence of oil, disclosing even the minutes leaks. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. Oil serves two functions : i) Keeps the coil cool by circulation and   ii) Provides the transformers an additional insulation.

The oil should be absolutely free from alkalies, sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly. Hence the tanks are sealed air tight to avoid the contact of oil with atmospheric air and moisture. In large transformers, the chambers called breather are provided. The breathers prevent the atmospheric moisture to pass on to the oil. The breathers contain the silica gel crystal which immediately absorb the atmospheric moisture. Due to long and continuous use, the sludge is formed in the oil which can contaminate the oil. Hence to keep such sludge separate from the oil in main tank, an air tight metal drum is provided, which is placed on the top of tank. This is called conservator.

Comparison of Core and Shell Type Transformers

Sr. No.	Core Type	Shell Type
1.	The winding encircles the core.	The core encircles most part of the windings.
2.	The cylindrical type of coils are used.	Generally, multilayer disc type or sandwich coils are used.
3.	As windings are distributed, the natural cooling is more effective.	As windings are surrounded by the core, the natural cooling does not exist.
4.	The coils can be easily removed from maintenance point of view.	For removing any winding for the maintenance, large number of laminations are required to be removed. This is difficult.
5.	The construction is preferred for low voltage transformers.	The construction is used for very high voltage transformers.
6.	It has a single magnetic circuit.	It has a double magnetic circuit.
7.	In a single phase type, the core has two limbs.	In a single phase type, the core has three limbs.

E.M.F EQUATION OF TRANSFORMER:

When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux Φ. The primary winding has N_1 number of turns. The alternating flux Φ linking with the primary winding itself induces an e.m.f in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of Φ_m as show in the Fig. 1.

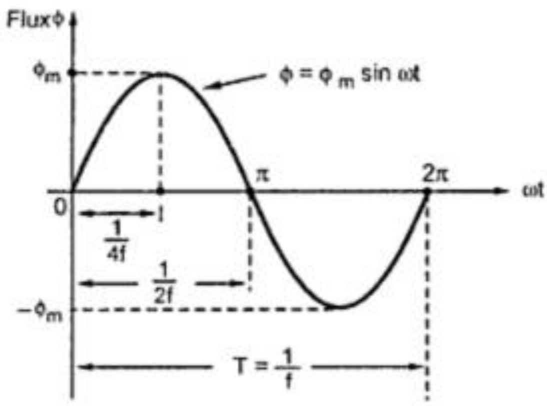


Fig. 1 Sinusoidal flux

The various quantities which affect the magnitude of the induced e.m.f. are :

Φ = Flux

Φ_m = Maximum value of flux

N_1 = Number of primary winding turns

N_2 = Number of secondary winding turns

f = Frequency of the supply voltage

E_1 = R.M.S. value of the primary induced e.m.f.

E_2 = R.M.S. value of the secondary induced e.m.f.

From Faraday's law of electromagnetic induction the voltage e.m.f. induced in each turn is proportional to the average rate of change of flux.

$\therefore$ average e.m.f. per turn = average rate of change of flux

$\therefore$ average e.m.f. per turn = $d\Phi/dt$

Now $d\Phi/dt$ = Change in flux/Time required for change in flux

Consider the 1/4 th cycle of the flux as shown in the Fig.1. Complete cycle gets completed in 1/f seconds. In 1/4 th time period, the change in flux is from 0 to Φ_m.

$\therefore$ $d\Phi/dt = (\Phi_m - 0)/(1/4f)$ as dt for 1/4 th time period is 1/4f seconds

$= 4\ f\ \Phi_m$ Wb/sec

$\therefore$ Average e.m.f. per turn = $4\ f\ \Phi_m$ volts

As is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity,

From factor = R.M.S. value/Average value = 1.11

$\therefore$ R.M.S. value of induced e.m.f. per turn

$= 1.11 \times 4\ f\ \Phi_m = 4.44\ f\ \Phi_m$

There are number of primary turns hence the R.M.S value of induced e.m.f. of primary denoted as is E_1,

$E_1 = N_1 \times 4.44\ f\ \Phi_m$ volts

While as there are number of secondary turns the R.M.S values of induced e.m.f. of secondary denoted is E_2 is,

$E_2 = N_2 \times 4.44\ f\ \Phi_m$ volts

The expression of E_1 and E_2 are called e.m.f. equation of a transformer.

Thus e.m.f. equations are,

$E_1 = 4.44\ f\ \Phi_m\ N_1$ volts ....(1)

$E_2 = 4.44\ f\ \Phi_m\ N_2$ volts ....(2)

Transformation Ratio(k)

Consider a transformer shown in Fig.1 indicating various voltages and currents.

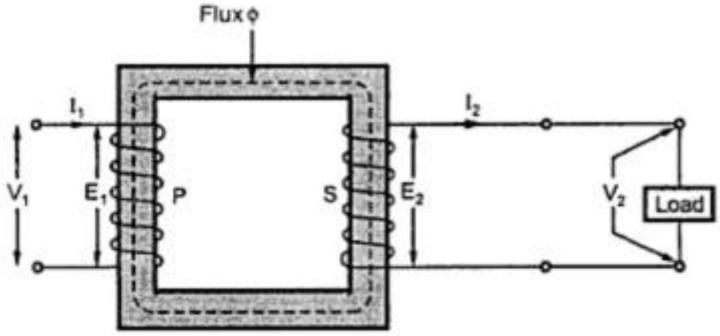


Fig. 1 Ratios of transformer

1. Voltage Ratio

We known from the e.m.f. equations of a transformer that

$E_1 = 4.44 \, f \, \Phi_m \, N_1$ and $E_2 = 4.44 \, f \, \Phi_m \, N_2$

Taking ratio of the two equations we get,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This ratio of secondary induced e.m.f. to primary induced e.m.f. is known as voltage transformation ratio denoted as K,

Thus,

$$E_2 = K E_1 \qquad \text{where} \quad K = \frac{N_2}{N_1}$$

1. If $N_2 > N_1$ i.e. $K > 1$, $E_2 > E_1$ we get then the transformer is called step-up transformer.
2. If $N_2 < N_1$ i.e. $K < 1$, we get $E_2 < E_1$ then the transformer is called step-down transformer.
3. If = i.e. K= 1, we get $E_2 = E_1$ then the transformer is called isolation transformer or 1:1 transformer.

2. Concept of Ideal Transformer

A transformer is said to be ideal if it satisfies following properties :

i) It has no losses.

ii) Its windings have zero resistance.

iii) Leakage flux is zero i.e. 100% flux produced by primary links with the secondary.

iv) Permeability of core is so high that negligible current is required to establish the flux in it.

Key point : For an ideal transformer, the primary applied voltage V_1 is same as the primary induced e.m.f. V_2 as there are no voltage drops.

Similarly the secondary induced e.m.f. E_2 is also same as the terminal voltage V_2 across the load. Hence for an ideal transformer we can write,

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = K$$

No transformer is ideal in practice but the value of E_1 is almost equal to V_1 for properly designed transformer.

3. Current ratio

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So $V_1 I_1$ = input VA and $V_2 I_2$ = output VA

For an ideal transformer,

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Key point : Hence the currents are in the inverse ratio of the voltage transformation ratio.

4. Voltage ampere rating

When electrical power is transferred from primary winding to secondary there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device.Now this temperature must be maintained below certain limiting values as it is always harmful from insulation point of view. As current is the main cause in producing heat, the output maximum rating is generally specified as the product of output voltage and output current i.e.$V_2 I_2$. This always indicates that when transformer is operated under this specified rating, its temperature rise will not be excessive. The copper loss (I^2R) in the transformer depends on the current 'I' through the winding while the iron or core loss depends on the voltage 'V' as frequency of operation is constant. None of these losses depend on the power factor (cos Φ) of the load. Hence losses decide the temperature and hence the rating of the transformer. As losses depend on V and I only, the rating of the transformer is specified as a product of these two parameters VxI.

Key point : Thus the transformer rating is specified as the product of voltage and current called VA rating.

On both sides, primary and secondary VA rating remains same. This rating is generally expresses in KVA (kilo volt amperes rating).

Now $V_1/V_2 = I_2/I_1 = K$

$\therefore$ $V_1 I_1 = V_2 I_2$

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

If V_1 and V_2 are the terminal voltages of primary and secondary then from specified KVA rating we can decide full load currents of primary and secondary, I_1 and I_2. This is the safe maximum current limit which may carry, keeping temperature rise below its limiting value.

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Key point : The full load primary and secondary currents indicate the safe maximum values of currents which transformer windings can carry.

Example 1 : A single phase, 50 Hz transformer has 80 turns on the primary winding and 400 turns on the secondary winding. The net cross-sectional area of the core is 200 cm^2. If the primary winding is connected at a 240 V , 50 Hz supply, determine :

i) The e.m.f. induced in the secondary winding.
ii) The maximum value of the flux density in the core.

Solution

$N_1 = 80$, $f = 50$ Hz , $N_2 = 400$, $a = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ cm}^2$

$E_1 = 240$

$K = N_2 / N_1 = 400/80 = 5/1$

$\therefore \quad K = E_2 / E_1 = E_2 /240 = 5/1$

$E_2 = 5 \times 240 = 1200$ V

Now $\quad E_1 = 4.44 \, f \, \Phi_m N_1$

$240 = 4.44 \times 50 \times \Phi_m \times 80$

$\therefore \quad \Phi_m = 240/(4.44 \times 50 \times 80) = 0.01351$ Wb

$\therefore \quad B_m = \Phi_m /a = 0.01351/(200 \times 10^{-4}) = 0.6756 \text{ Wb/m}^2$

Example 2 : For a single phase transformer having primary and secondary turns of 440 and 880 respectively, determine the transformer KVA rating if half load secondary current is 7.5 A and maximum value of core flux is 2.25 Wb.

Solution

$N_1 = 440$, $\quad N_2 = 880$, $\quad (I_2)_{H.L.} = 7.5$ A,

$f_m = 2.25$ mWb , $\quad E_2 = 4.44 \, \Phi_m f \, N_2$

Assuming $\quad f = 50$ Hz,

$\therefore \quad E_2 = 4.44 \times 2.25 \times 10^{-3} \times 50 \times 880 = 439.56$ V

$(I_2)_{F.L.} = \text{KVA rating} / E_2$

And $\quad (I_2)_{H.L.} = 0.5 \, (I_2)_{F.L.}$

$\therefore \quad (I_2)_{H.L.} = 0.5 \times (\text{KVA rating} / E_2)$

$\therefore \quad 7.5 = 0.5 \times (\text{KVA rating} / 439.56)$

$\therefore$ KVA rating $= 2 \times 7.5 \times 439.56 \times 10^{-3}$

$= 6.5934$ KVA .....(10^{-3} for KVA)

Example 3 : A single phase transformer has 350 primary and 1050 secondary turns. The primary is connected to 400 V, 50 Hz a.c. supply. If the net cross-sectional area of the core is 50 cm^2, calculate i) The maximum value of the flux density in the core ii) The induced e.m.f. in the secondary winding.

Solution

The given value are,

N_1 = 350 turns, N_2 = 1050 turns

V_1 = 400 V , A = 50 cm^2= 50 x 10^{-4} m^2

The e.m.f. of the transformer is,

$E_1 = 4.44\ f\ \Phi_m N_1$

$E_1 = 4.44\ B_m A\ f\ N_1$ as $\Phi_m = B_m A$

Flux density $B_m = E_1 / (4.44\ A\ f\ N_1)$

$= 400 / (4.44 \times 50 \times 10^{-4} \times 50 \times 350)$ assume $E_1 = V_1$

$= 1.0296\ Wb/m^2$

$K = N_2/N_1 = 1050/350 = 3$

And $K = E_2/E_1 = 3$

∴ $E_2 = 3 \times E_1 = 3 \times 400 = 1200$ V

IDEAL TRANSFORMER ON NO-LOAD:

Consider an ideal transformer on no load as shown in the Fig. 3. The supply voltage is and as it is V_1 an no load the secondary current $I_2 = 0$.

The primary draws a current I_1 which is just necessary to produce flux in the core. As it magnetising the core, it is called magnetising current denoted as I_m. As the transformer is ideal, the winding resistance is zero and it is purely inductive in nature. The magnetising current is I_m is very small and lags V_1 by 30^o as the winding is purely inductive. This I_m produces an alternating flux Φ which is in phase with I_m.

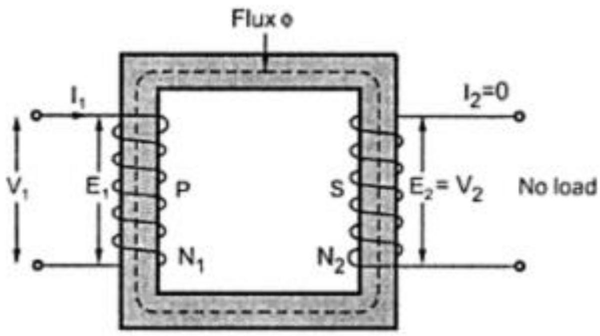


Fig. 1 Ideal transformer on no load

The flux links with both the winding producing the induced e.m.f.s E_1 and E_2 , in the primary and secondary windings respectively. According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence E_1 is in antiphase with V_1 but equal in magnitude. The induced E_2 also opposes V_1 hence in antiphase with V_1 but its magnitude depends on N_2. Thus E_1 and E_2 are in phase.

The phasor diagram for the ideal transformer on no load is shown in the Fig. .2.

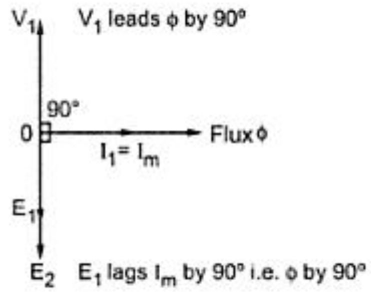


Fig. 2 Phasor diagram for ideal transformer on no load

It can be seen that flux Φ is reference. I_m produces Φ hence in phase with Φ. V_1 leads I_m by 90^o as winding is purely inductive so current has to lag voltage by 90^o.

E_1 and E_2 are in phase and both opposing supply voltage .

The power input to the transformer is $V_1 I_1 \cos (V_1 \wedge I_1)$ i.e. $V_1 I_m \cos(90^o)$ i.e. zero. This is because on no load output power is zero and for ideal transformer there are no losses hence input power is also zero. Ideal no load p.f. of transformer is zero lagging.

PRACTICAL TRANSFORMER ON NO-LOAD:

Actually in practical transformer iron core causes hysteresis and eddy current losses as it is subjected to alternating flux. While designing the transformer the efforts are made to keep these losses minimum by,

1. Using high grade material as silicon steel to reduce hysteresis loss.
2. Manufacturing core in the form of laminations or stacks of thin lamination to reduce eddy current loss.

Apart from this there are iron losses in the practical transformer. Practically primary winding has certain resistance hence there are small primary copper loss present.

Thus the primary current under no load condition has to supply the iron losses i.e. hysteresis loss and eddy current loss and a small amount of primary copper loss. This current is denoted as I_o.

Now the no load input current I_o has two components :

1. A purely reactive component I_m called magnetising component of no load current required to produce the flux. This is also called wattless component.
2. An active component I_c which supplies total losses under no load condition called power component of no load current. This also called wattful component or core loss component of I_o.

Th total no load current I_o is the vector addition of I_m and I_c.

$$\boxed{\bar{I}_o = \bar{I}_m + \bar{I}_c} \quad \dots (1)$$

In practical transformer, due to winding resistance, no load current I_o is no longer at 90^o with respect to V_1. But it lags V_1 by angle Φ_o which is less than 90^o . Thus cos Φ_o is called no load power factor of practical transformer.

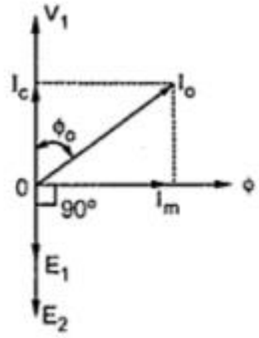


Fig 1. Practical transformer on no load

The phasor diagram is shown in the Fig. 1. It can be seen that the two components I_o are,

$$\boxed{I_m = I_o \sin\phi_o} \quad \ldots (2)$$

This is magnetising component lagging V_1 exactly by 90^o .

$$\boxed{I_c = I_o \cos\phi_o} \quad \ldots (3)$$

This is core loss component which is in phase withV_1.
The magnitude of the no load current is given by,

$$\boxed{I_o = \sqrt{I_m^2 + I_c^2}} \quad \ldots (4)$$

While Φ_o = no load primary power factor angle
The total power input on no load is denoted as W_o and is given by,

$$\boxed{W_o = V_1 I_o \cos\phi_o = V_1 I_c} \quad \ldots (5)$$

It may be denoted that the current is very small, about 3 to 5% of the full load rated current. Hence the primary copper loss is negligibly small hence I_c is called core loss or iron loss component. Hence power input W_o on no load always represent the iron losses, as copper loss is negligibly small. The iron losses are denoted as P_i and are constant for all load conditions.

$$\therefore \quad \boxed{W_o = V_1 I_o \cos\phi_o = P_i = \text{iron loss}} \quad \ldots (6)$$

Example 1 : The no load current of a transformer is 10 A at a power factor 0f 0.25 lagging, when connected to 400 V, 50 Hz supply. Calculate,

a) Magnetising component of the no load current
b) Iron loss and c) Maximum value of flux in the core.
Assume primary winding turns as 500.

Solution : The given value are, = 10 A, cos = 0.25, = 400 V and f = 50 Hz

a) $I_m = I_o \sin \Phi_o$ = magnetising component

$\Phi_o = \cos^{-1}(0.25) = 75.522°$

∴ $I_m = 10 \times \sin(75.522°) = 9.6824$ A

b) P_i = iron loss = power input on no load

$= W_o = V_1 I_o \cos \Phi_o = 400 \times 10 \times 0.25$

= 1000 W

c) On no load, $E_1 = V_1 = 400$ V and $N_1 = 500$

Now $E_1 = 4.44 f \Phi_m N_1$

∴ $400 = 4.44 \times 50 \times \Phi_m \times 500$

∴ $\Phi_m = 3.6036$ mWb

TRANSFORMER ON LOAD (M.M.F Balancing on Load)

When the transformer is loaded, the current I_2 flows through the secondary winding. The magnetic and phase of I_2 is determined by the load. If load is inductive, I_2 lags V_2. If load is capacitive, I_2 leads V_2 while for resistive load, I_2 is in phase with V_2.

There exists a secondary m.m.f. $N_2 I_2$ due to which secondary current sets up its own flux Φ_2. This flux opposes the main flux Φ which is produced in the core due to magnetising component of no load current. Hence the m.m.f. is $N_2 I_2$ called demagnetising ampere-turns. This is shown in the Fig.1(a).

The flux Φ_2 momentarily reduces the main flux Φ, due to which the primary induced e.m.f. also E_1 reduces.

Hence the vector difference $\bar{V} - \bar{E}$ increases due to which primary draws more current from supply

This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Fig.1(b).

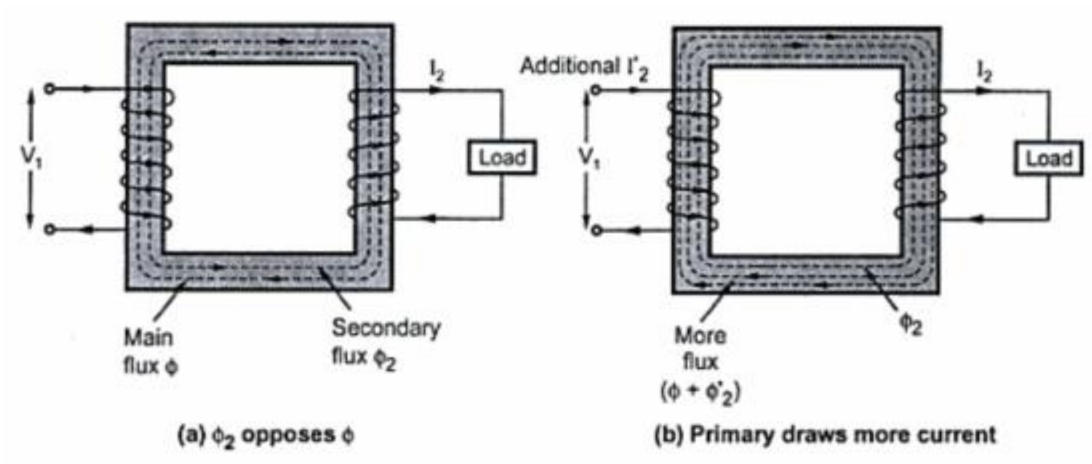


Fig. 1 Transformer on load

This current I_2' is in antiphase with I_2. The current sets up its own flux Φ_2' which opposes the flux Φ_2 and helps the main flux Φ. This flux Φ_2' neutralises the flux Φ_2 produced by I_2. The m.m.f. i.e. ampere turns $N_2 I_2'$ balances the ampere turns $N_2 I_2$. Hence the net flux in the core is again maintained at constant level.

Key point : Thus for any load condition, no load to full load the flux in the core is practically constant.

The load component current I_2' always neutralises the changes in the loads. Hence the transformer is called constant flux machine.

As the ampere turns are balanced we can write,

$$N_2 I_2 = N_2 I_2'$$

$$\therefore \quad I_2' = (N_2/N_1) = K\, I_2 \qquad ....(1)$$

Thus when transformer is loaded, the primary current I_1 has two components :

1. The no load current I_o which lags V_1 by angle Φ_o. It has two components I_m and I_c.

2. The load component I_2' which in antiphase with I_2. And phase of I_2 is decided by the load.

Hence primary current I_1 is vector sum of I_o and I_2'.

$$\therefore \quad \bar{I}_1 = \bar{I}_o + \bar{I}_2 \qquad ....(2)$$

Assume inductive load, I_2 lags E_2 by Φ_2, the phasor diagram is shown in the Fig. 2(a).

Assume purely resistive load, I_2 in phase with E_2, the phasor diagram is shown in the Fig.2(b).

Assume capacitive load, I_2 leads E_2 by Φ_2, the phasor diagram is shown in the Fig. 2(c).

Note that I_2' is always in antiphase with I_2.

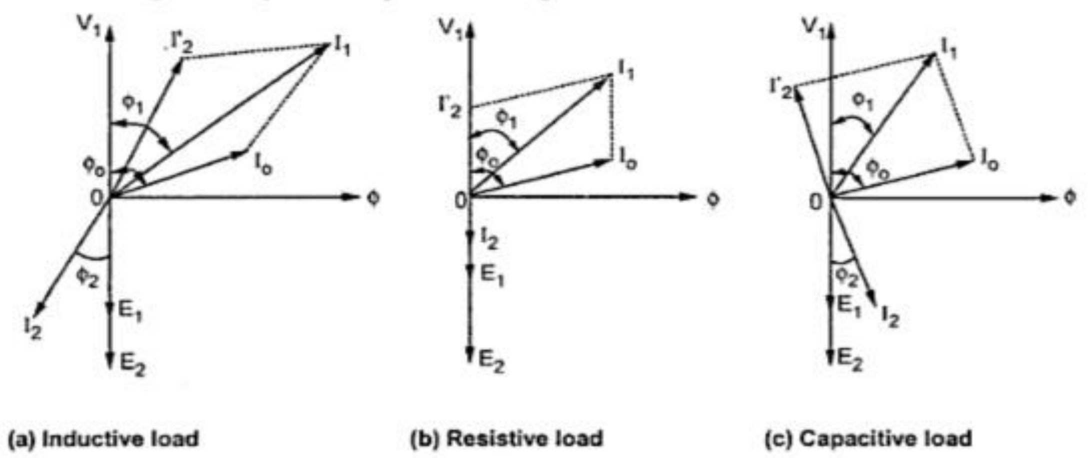


Fig. 2

Actually the phase of I_2 is with respect to V_2 i.e. angle Φ_2 is angle between I_2 and V_2. For the ideal case, E_2 is assumed equal to V_2 neglecting various drops.

The current ratio can be verified from this discussion. As the no load current I_o is very small, neglecting I_o we can write,

$$I_1 \simeq I_2'$$

Balancing the ampere turns,

$$N_1 I_1 = N_1 I_1 = N_2 I_2$$

$$\therefore \quad N_2/N_1 = I_1/I_2 = K$$

Under full load conditions when I_o is very small compared to full load currents, the ratio of primary and secondary current is constant.

Example : A 400/200 V transformer takes 1 A at a power factor of 0.4 on no load. If the secondary supplies a load current of 50 A at 0.8 lagging power factor, calculate the primary current.

Solution : The given values are

$I_o = 1$ A, $\cos \Phi_o = 0.4$, $I_2 = 50$ A and $\cos \Phi_2 = .08$

$K = E_2/E_1 = 200/400 = 0.5$

$\therefore \quad I_2' = K\, I_2 = 0.5 \times 50 = 25$ A

The angle of I_2' is to be decided from $\cos \Phi_2 = 0.8$

Now $\quad \cos \Phi_2 = 0.8$

$\therefore \quad \Phi_2 = 36.86^o$

I_2' is antiphase with I_2 which lags E_2 by 36.86^o

Consider the phasor diagram shown in the Fig. 3. The flux Φ is the reference.

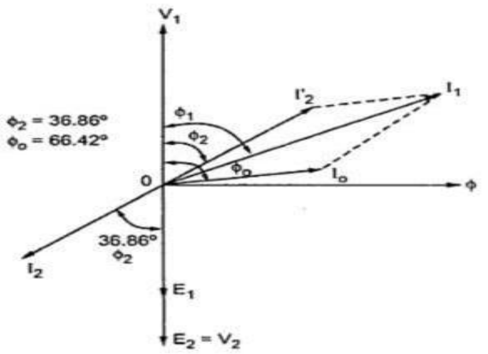


Fig. 3

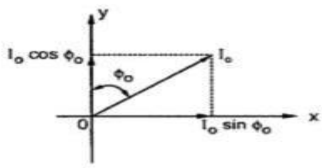


Fig. 3 (a)

Now $\quad \cos \Phi_o = 0.4$

$\therefore \quad \Phi_o = 66.42^o$

$\bar{I}_1 = \bar{I}_2' + \bar{I}_o$ .... vector sum

Resolve I_o and I_2' into two components, along reference Φ and in quadrature with Φ in phase with V_1.

x component of $I_o = I_o \sin \Phi_o = 0.9165$ A

y component $I_o = I_o \cos \Phi_o = 0.4$ A

$\therefore \quad \bar{I}_o = 0.9165 + j\, 0.4$ A

x component of $I_2' = I_2' \sin \Phi_2 = 25 \sin (36.86^o) = 15$ A

y component of $I_2' = I_2' \cos \Phi_2 = 25 \times 0.8 = 20$ A

$\therefore \quad I_2' = 15 + j\, 20$ A

$\bar{I}_1 = 0.9165 + j\, 0.4 + 15 + j\, 20 = 15.9165 + j\, 20.4$ A

Thus the two components of I_1 are as shown in the Fig.3(c).

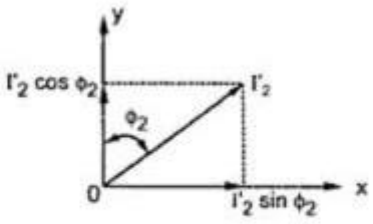


Fig. 3 (b)

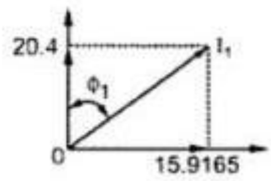


Fig. 3 (c)

∴ $I_1 = \sqrt{(15.9165)^2 + (20.4)^2} = 25.874$ A

This is the primary current magnitude.

While $\tan \Phi_1 = 15.9165/20.4$

∴ $\Phi_1 = 37.96^o$

Hence the primary power factor is,

$\cos \Phi_1 = \cos (37.96^o) = 0.788$ lagging

Key point : Remember that Φ_1 is angle between V_1 and I_1 and as V_1 is vertical, Φ_1 is measured with respect to V_1. So do not convert rectangular to polar as it gives angle with respect to x-axis and we want it with respect to y-axis.

Effect OF Winding Resistances

A practical transformer windings process some resistances which not only cause the power losses but also the voltage drops. Let us see what is the effect of winding resistance on the performance of the transformer.

Let R_1 = primary winding resistance in ohms

R_2 = secondary winding resistance in ohms

Now when current I_1 flows through primary, there is voltage drop $I_1 R_1$ across the winding. The supply voltage V_1 has to supply this drop. Hence primary induced e.m.f. E_1 is the vector difference between V_1 and $I_1 R_1$.

$$\therefore \quad \overline{E_1} = \overline{V_1} - \overline{I_1 R_1} \quad \ldots (1)$$

Similarly the induced e.m.f. in secondary is E_2. When load is connected, current I_2 flows and there is voltage drop $I_2 R_2$. The e.m.f. E_2 has to supply this drop. The vector difference between E_2 and $I_2 R_2$ is available to the load as a terminal voltage.

$$\therefore \quad \overline{V_2} = \overline{E_2} - \overline{I_2 R_2} \quad \ldots (2)$$

The drops $I_1 R_1$ and $I_2 R_2$ are purely resistive drops hence are always in phase with the respective currents I_1 and I_2.

Equivalent Resistance

The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer. The transfer of the resistances on any one side is advantageous as it makes the calculations very easy. Let us see how to transfer the resistances on any one side.

The total copper loss due to both the resistances can be obtained as,

total copper loss = $I_1^2 R_1 + I_2^2 R_2$

$= I_1^2\{ R_1 + (I_2^2/I_1^2) R_2\}$

$= I_1^2\{R_1 + (1/K^2) R_2\}$ ....(3)

Where $I_2/I_1 = 1/K$ neglecting no load current.

Now the expression (3) indicates that the total copper loss can be expressed as $I_1^2 R_1 + I_1^2 . R_2/K^2$. This means R_2/K^2 is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with. This value of resistance which R_2/K^2 is the value of R_2 referred to primary is called equivalent resistance of secondary referred to primary. It is denoted as R_2'.

$R_2' = R_2/K^2$ ....(4)

Hence the total resistance referred to primary is the addition of R_1 and R_2' called equivalent resistance of transformer referred to primary and denoted as R_{1e}.

$= R_1 + R_2' = R_1 + R_2/K^2$ ....(5)

This resistance R_{1e} causes same copper loss with I_1 as the total copper loss due to the individual windings.

total copper loss = $I_1^2 R_{1e} = I_1^2 R_1 + I_2^2 R_2$ ......(6)

So equivalent resistance simplifies the calculations as we have to calculate parameters on one side only.

Similarly it is possible to refer the equivalent resistance to secondary winding.

total copper loss = $I_1^2 R_1 + I_2^2 R_2$

$= I_2^2 \{(I_1^2/I_2^2) R_1 + R_2\}$

$= I_2^2 (K^2 R_1 + R_2)$ ....(7)

Thus the resistance $K^2 R_1$ is primary resistance referred to secondary denoted as R_1'.

$R_1' = K^2 R_1$ ....(8)

Hence the total resistance referred to secondary is the addition of R_2 and R_1' called equivalent resistance of transformer referred to secondary and denoted as R_{2e}.

$R_{2e} = R_2 + R_1' = R_2 + K^2 R_1$ ....(9)

total copper loss = $I_2^2 R_{2e}$ ....(10)

The concept of equivalent resistance is shown in the Fig. 1(a), (b) and (c).

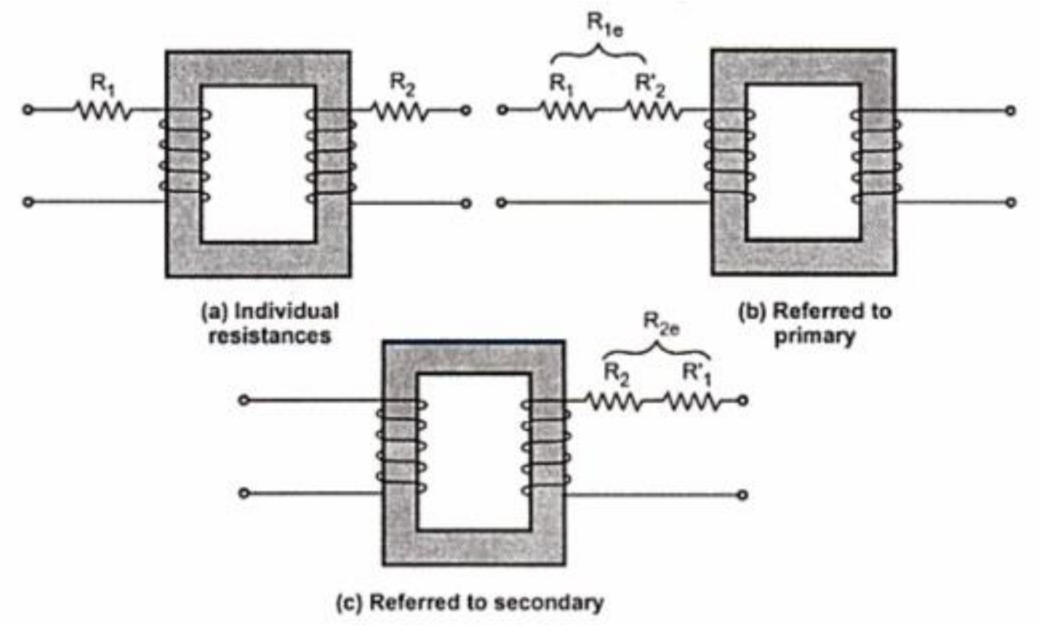


Fig. 1 Equivalent resistance

Key Point : When resistance are transferred to primary, the secondary winding becomes zero resistance winding for calculation purpose. The entire copper loss occurs due to R_{1e}. Similarly when resistances are referred to secondary, the primary becomes resistanceless for calculation purpose. The entire copper loss occurs due to R_{2e}.

Important Note : When a resistance is to be transferred from the primary to secondary, it must be multiplied by K^2. When a resistance is to be transferred from the secondary to primary, it must be divided by K^2. Remember that K is N_1/N_2.

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

Key point :
High voltage side → Low current side → High resistance side
Low voltage side → High current side → Low resistance side

Example 1 : A 6600/400 V single phase transformer has primary resistance of 2.5 Ω and secondary resistance of 0.01 Ω calculate total equivalent resistance referred to primary and secondary.

Solution : The given values are,

$R_1 = 2.5\ \Omega$ $\quad$ $R_2 = 0.01\ \Omega$

$K = 400/6600 = 0.0606$

While finding equivalent resistance referred to primary, transfer to primary as,

$R_2' = R_2/K^2 = 0.01/(0.0606)^2 = 2.7225\ \Omega$

$R_{1e} = R_1 + R_2' = 2.5 + 2.7225 = 5.2225\ \Omega$

It can be observed that primary is high voltage hence high resistance side hence while transferring from low voltage to on high voltage, its value increases.

To find total equivalent resistance referred to secondary, first calculate ,

$R_1' = K^2\ R_1 = (0.0606)^2 \times 25 = 0.00918\ \Omega$

$R_{2e} = R_2 + R_1' = 0.01 + 0.00918 = 0.01918\ \Omega$

Effect of Leakage Reactance

Uptill now it is assumed that the entire flux produced by the primary links with the secondary winding. But in practice it is not possible. Part of the primary flux as well as the secondary flux completes the path through air and links with the respecting winding only. Such a flux is called leakage flux. Thus there are two leakage fluxes present as shown in the Fig. 1.

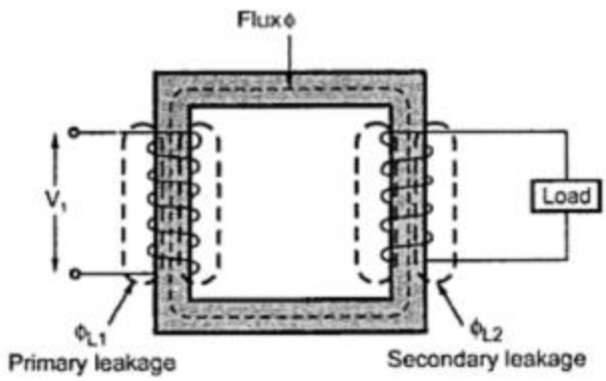


Fig .1 Individual impedance

The flux Φ_{L1} is the primary leakage flux which is produced due to primary current I_1. It is in phase with I_1 and links with primary only.

The flux Φ_{L2} is the secondary leakage flux which is produced due to current I_2. It is in phase with I_2 and links with the secondary winding only.

Due to the leakage flux Φ_{L1} there is self induced e.m.f. e_{L1} in primary. While due to leakage flux Φ_{L2} there is self induced e.m.f. e_{L2} in secondary. The primary voltage V_1 has to overcome this voltage e_{L1} to produce E_1 while induced e.m.f. E_2 has to overcome e_{L2} to produce terminal voltage V_2. Thus the self induced e.m.f.s are treated as the voltage drops across the fictitious reactance placed in series with the windings. These reactances are called leakage reactance of the winding.

So X_1 = Leakage reacatnce of primary winding.

and X_2 = Leakage reactance of secondary winding.

The value of X_1 is such that the drop $I_1\ X_1$ is nothing but the self induced e.m.f. e_{L1} due to fluxΦ_{L1}. The value of X_2 is such that the drop $I_2\ X_2$ is equal to the self induced e.m.f. e_{L2} due to flux Φ_{L1}.

Leakage fluxes with the respective windings only and not to both the windings. To reduce the leakage, as mentioned, int eh construction both the winding's are placed on same limb rather than on separate limbs.

Equivalent Leakage Reactance

Similar to the resistances, the leakage reactances also can be transferred from primary to secondary or viceversa. The relation through K^2 remains same for the transfer of recatnaces as it is studied earlier for the resistances.

Let X_1 is leakage reactance of primary and X_2 is leakage reactance of secondary.

Then the total leakage reacatance referred to primary is X_{1e} given by,

$$X_{1e} = X_1 + X_2' \text{ where } \quad X_2' = X_2/K^2$$

While the total leakage reacatnce referred to secondary is given by ,

$$X_{2e} = X_2 + X_1' \text{ where } X_1' = K^2 X_1$$

And $K = N_2/N_1$ =transformation ratio

Equivalent Impedance

The transformer primary has resistance R_1 and reactance X_1. While the transformer secondary has resistance R_2 and reacatnce X_2. Thus we can say that the total impedance of primary winding is Z_1 which is,

$$Z_1 = R_1 + j X_1 \ \Omega \qquad \text{..........(1)}$$

And the total impedance of the secondary winding is which is ,

$$Z_2 = R_2 + j X_2 \ \Omega \qquad \text{...........(2)}$$

This is shown in the Fig. 1.

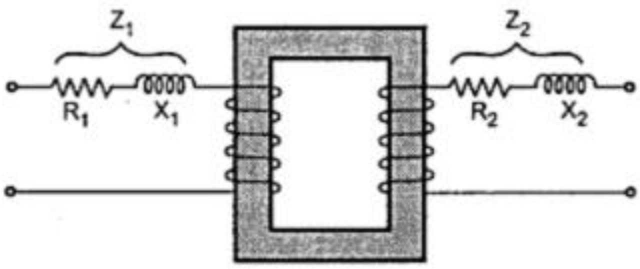


Fig. 1 Individual impedance

The individual magnitudes of and are,

$$Z_1 = \sqrt{(R_1^2 + X_1^2)} \qquad \text{...........(3)}$$

and $$Z_2 = \sqrt{(R_2^2 + X_2^2)} \qquad \text{.........(4)}$$

Similar to resistance and reactance, the impedance also can be referred to any one side.

Let Z_{1e} = total equivalent impedance referred to primary then $Z_{1e} = R_{1e} + j X_{1e}$

$$Z_{1e} = Z_1 + Z_2' = Z_1 + Z_2/K^2 \quad \text{...........(5)}$$

Similarly Z_{2e} = total equivalent impedance referred to secondary

then $Z_{2e} = R_{2e} + j X_{2e}$

$$Z_{2e} = Z_2 + Z_1' = Z_2 + K^2 Z_1 \quad \text{...........(6)}$$

The magnitude of Z_{1e} and Z_{2e} are,

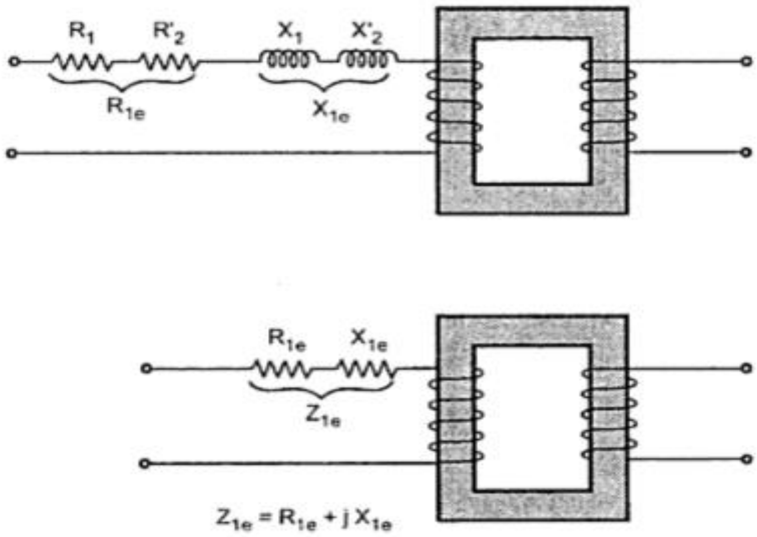
$$Z_{1e} = \sqrt{(R_{1e}^2 + X_{1e}^2)} \quad \text{........(7)}$$

and $$Z_{2e} = \sqrt{(R_{2e}^2 + X_{2e}^2)} \quad \text{..........(8)}$$

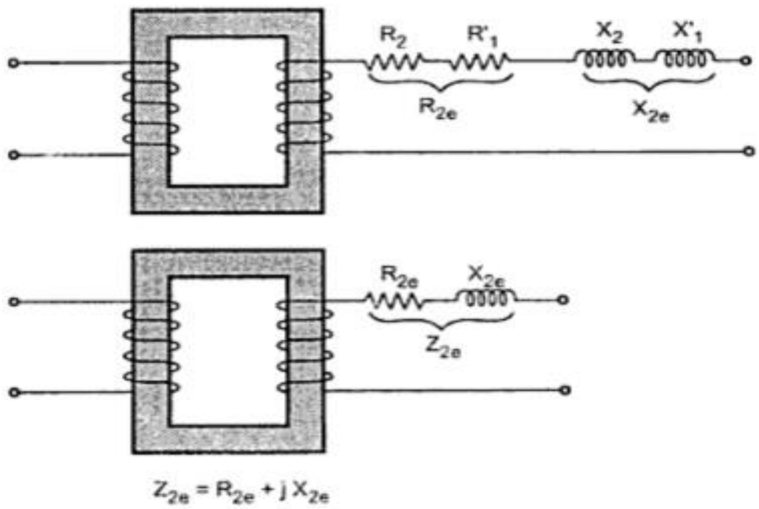
It can be denoted that,

$$Z_{2e} = K^2 Z_{1e} \quad \text{and} \quad Z_{1e} = Z_{2e} / K^2$$

The concept of equivalent impedance is shown in the Fig. 2.



(a) Referred to primary



(b) Referred to secondary

Fig 2 Equivalent impedance

Example 1 :A 15 KVA, 2200/110 V transformer has $R_1 = 1.75\Omega$, $R_2 = 0.0045\ \Omega$ the leakage reactance are $X_1 = 2.6\ \Omega$ and $X_2 = 0.0075\ \Omega$ Calculate,

a) equivalent resistance referred to primary

b) equivalent resistance referred to secondary

c) equivalent reactance referred to primary

d) equivalent reactance referred tosecondary

e) equivalent impedance referred to primary

f) equivalent impedance referred to secondary

g) total copper loss

Solution : The given values are, $R_1 = 1.75\ \Omega$, $R_2 = 0.0045\Omega$, $X_1 = 2.6\ \Omega$, $X_2 = 0.0075\ \Omega$

$K = 110/2200 = 1/20 = 0.05$

a) $R_{1e} = R_1 + R_2' = R_1 + R_2/K^2 = 1.75 + 0.0045/0.05^2 = 3.55\ \Omega$

b) $R_{2e} = R_2 + R_1' = R_2 + K^2\ R_1 =$

$= 0.0045 + (0.05)^2 \times 1.75 = 0.00887\ \Omega$

c) $X_{1e} = X_1 + X_2' = X_1 + X_2/K^2 = 2.6 + 0.0075/(0.05)^2 = 5.6\ \Omega$

d) $X_{2e} = X_2 + X_1' = X_2 + K^2\ X_1$

$= 0.0075 + (0.05)^2 \times 2.6 = 0.014\ \Omega$

e) $Z_{1e} = R_{1e} + j\ X_{1e} = 3.55 + j\ 5.6\ \Omega$

$Z_{1e} = \sqrt{(3.55^2 + 5.6^2)} = 6.6304\ \Omega$

f) $Z_{2e} = R_{2e} + j\ X_{2e} = 0.00887 + j\ 0.014\ \Omega$

$Z_{2e} = \sqrt{(0.00887^2 + 0.014^2)} = 0.01657\ \Omega$

g) To find the load copper loss, calculate full load current.

(I_1) F.L. = (KVA x 1000)/V_1 = (25 x 1000)/2200 = 11.3636 A

total copper loss = $((I_1)F.L.)^2\ R_{1e} = (11.3636)^2 \times 355 = 458.4194$ W

This can be checked as,

(I_2) F.L.= (KVA x 1000)/V_2 = (25 x 1000/110 = 227.272 A

total copper loss = $I_1^2\ R_1 + I_2^2\ R_2$

$= (11.3636)^2 \times 1.75 + (227.373)^2 \times 0.0045$

$= 225.98 + 232.4365 = 458.419$ W

Equivalent circuit of Transformer

The term equivalent circuit of a machine means the combination of fixed and variable resistances and reactances, which exactly simulates performance and working of the machine.

For a transformer, no load primary current has two components,

$I_m = I_o \sin\Phi_o$ = Magnetizing component

$I_c = I_o \cos\Phi_o$ = Active component

I_m produces the flux and is assumed to flow through reactance X_o called no load reractance while I_c is active component representing core losses hence is assumed to flow through the reactance R_o. Hence equivalent circuit on no load can be shown as in the Fig. 1. This circuit consisting of R_o and X_o in parallel is called exciting circuit. From the equivalent circuit we can write,

$R_o = V_1/I_c$

and $X_o = V_1/I_m$

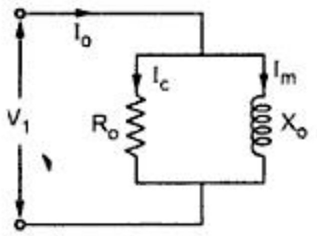


Fig. 1 No load equivalent circuit

When the is connected to the transformer then secondary current I_2 flows. This causes voltage drop across R_2 and R_2. Due to I_2, primary draws an additional current

$I_2' = I_2/K$. Now I_1 is the phasor addition of I_o and I_2'. This I_1 causes the voltage drop across primary resistance R_1 and reactance X_1.

Hence the equivalent circuit can be shown as in the Fig. 2.

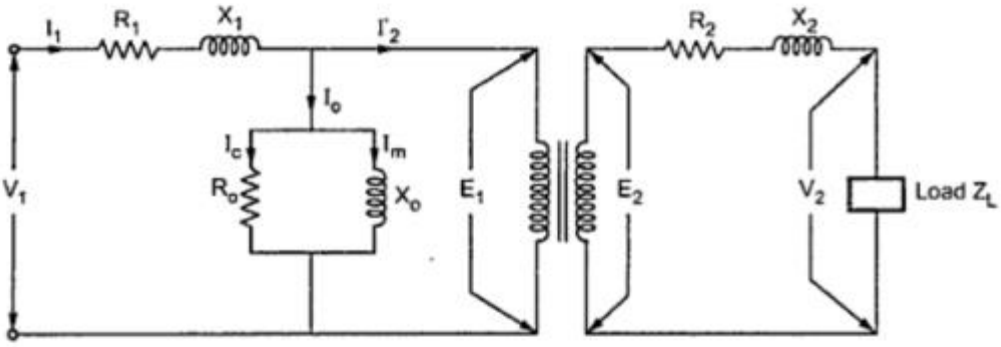


Fig. 2

But in the equivalent circuit, windings are not shown and it is further simplified by transferring all the values to the primary or secondary. This makes the transformer calculation much easy.

So transferring secondary parameters to primary we get,

$R_2' = R_2/K^2$, $X_2' = X_2/K^{2'}$, $Z_2' = Z_2/K^2$

While $E_2' = E_2/K'$ $I_2' = K I_2$

Where $K = N_2/N_1$

While transferring the values remember the rule that

Low voltage winding High current Low impedance

High voltage winding Low current High impedance

Thus the exact equivalent circuit referred to primary can be shown as in the Fig. 3.

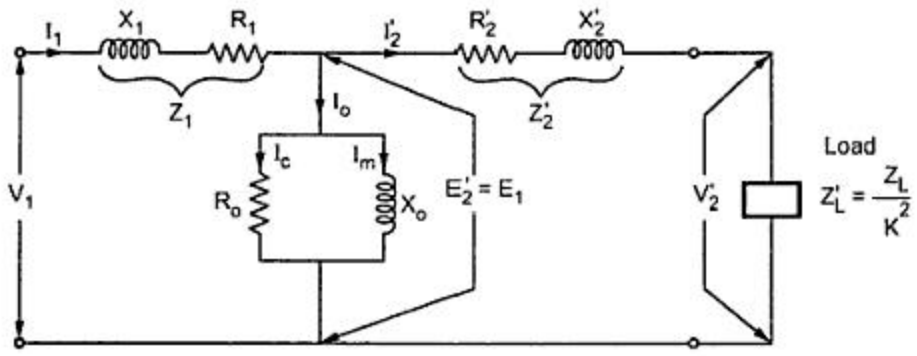


Fig. 3 Exact equivalent circuit referred to primary

Similarly all the primary value can be referred to secondary and we can obtain the equivalent circuit referred to secondary.

$R_1' = K^2 R_1$, $X_1' = K^2 X_1$, $Z_1' = K^2 Z_1$

$E_1' = K E_1$, $I_o' = I_1/K'$ $I_o' = I_o/K$

Similarly the exciting circuit parameters also gets transferred to secondary as R_o' and X_o'. The circuit is shown in the Fig.4.

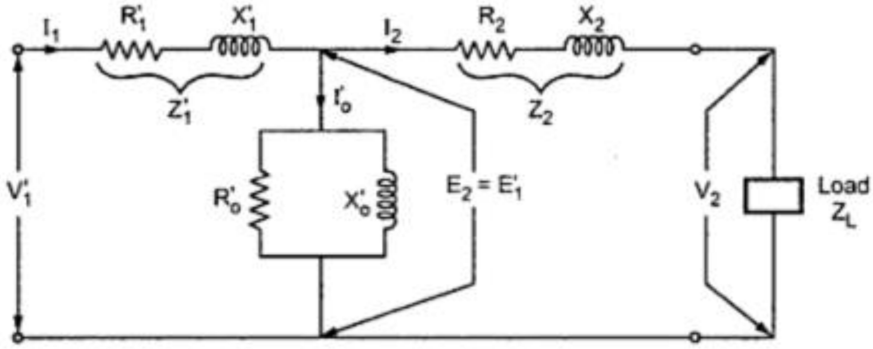


Fig. 4 Exact equivalent circuit referred to secondary

Now as long as no load branch i.e. exciting branch is in between Z_1 and Z_2', the impedances can not be combined. So further simplification of the circuit can be done. Such circuit is called approximate equivalent circuit.

<u>Approximate Equivalent Circuit</u>

To get approximate equivalent circuit, shift the no load branch containing R_o and X_o to the left of R_1 and X_1. By doing this we are creating an error that the drop across R_1 and X_1due to I_o is neglected. Hence such an equivalent circuit is called approximate equivalent circuit.

So approximate equivalent circuit referred to primary can be as shown in the Fig. 5.

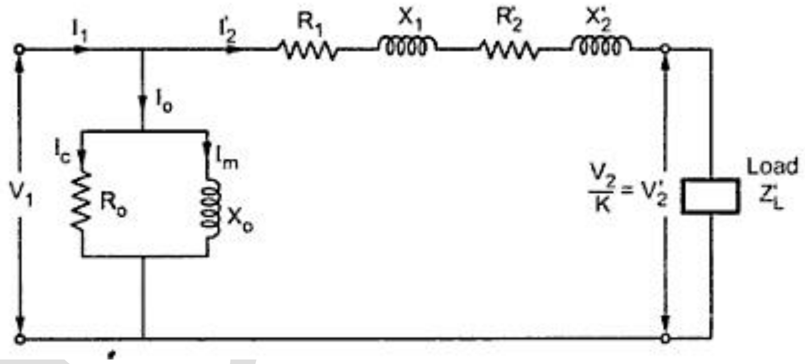


Fig. 5 Approximate equivalent circuit referred to primary

In this circuit now R_1 and R_2' can be combined to get equivalent resistance referred to primary R_{1e} as discussed earlier. Similarly $X_1$and X_1' can be combined to get X_{1e}. And equivalent circuit can be simplified as shown in the Fig. 6.

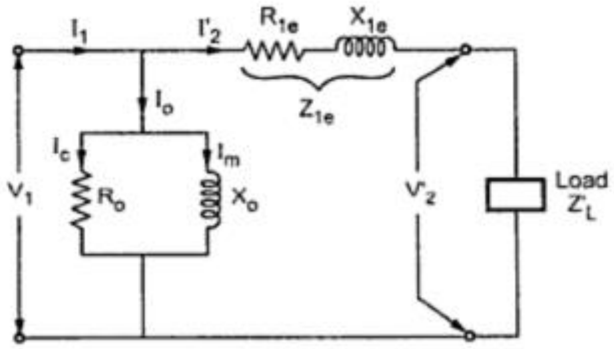


Fig. 6

We know that, $R_{1e} = R_1 + R_2' = R_1 + R_2/K^2$

$X_{1e} = X_1 + X_2' = X_1 + X_2/K^2$

$Z_{1e} = R_{1e} + j X_{1e}$

$R_o = V_1/I_c$ and $X_o = V_1/I_m$

$I_c = I_o \cos\Phi_o$ and $Im = I_o \sin\Phi_o$

In the similar fashion, the approximate equivalent circuit referred to secondary also can be obtained.

Approximate Voltage Drop in Transformer

Consider the equivalent circuit referred to secondary as shown in the Fig. 1.

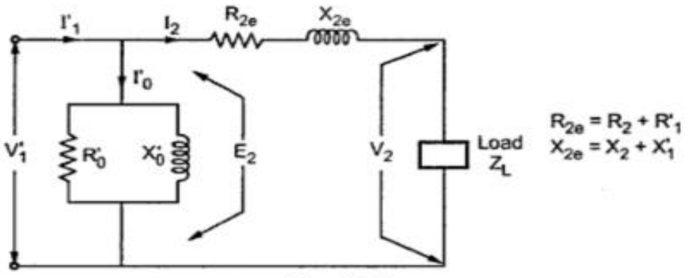


Fig. 1.33

Fig. 1

From the Fig. 1 we can write,

$$\bar{E}_2 = \overline{I_2 R_{2e}} + \overline{I_2 X_{2e}} + \bar{V}_2 = \bar{V}_2 + \bar{I}_2 (R_{2e} + j X_{2e})$$

$$\therefore \quad \bar{E}_2 = \bar{V}_2 + \overline{I_2 Z_{2e}}$$

As primary parameters are referred to secondary, there are no voltage drops in primary.

When there is no load, $I_2 = 0$ and we get no load terminal voltage V_{20} as E_2.

$\therefore$ $V_{20} = E_2$ = No load terminal voltage

while V_2 = Terminal voltage on load

Consider the phasor diagram for lagging p.f. load. The current I_2 lags V_2 by angle Φ_2. Take V_2 as reference phasor. $I_2 R_{2e}$ is in phase with I_2 while $I_2 X_{2e}$ leads I_2 by 90°. The phasor diagram is shown in the Fig.2.

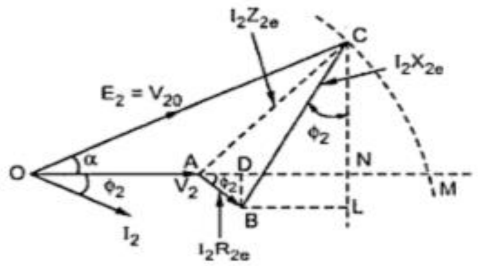


Fig. 2

To derive the expression for approximate voltage drop, draw the circle with O as centre and OC as redius, cutting extended OA at M. As OA = V_2 and now OM = E_2, the total voltage drop is AM = $I_2 Z_{2e}$.

But approximating this voltage drop is equal to AN instead of AM where N is intersection of perpendicular drawn from C on AM. This is because angle is practically very very small and in practice M and N are very close to each other.

Approximate voltage drop = AN

Draw perpendicular from B on AM intersecting it at D and draw parallel to DN from B to the point L shown in the Fig. 2.

$\therefore$ AD = AB cos Φ_2= $I_2 R_{2e}$ cos Φ_2

and DN = BL = BC sin Φ_2 = $I_2 X_{2e}$ sin Φ_2

$\therefore$ AN = AD + DN = $I_2 R_{2e} \cos \Phi_2 + I_2 X_{2e} \sin \Phi_2$

Assuming $\Phi_2 = \Phi_1 = \Phi$

$\therefore$ Approximate voltage drop = $I_2 R_{2e} \cos \Phi + I_2 X_{2e} \sin \Phi$

If all the parameters are referred to primary then we get,

Approximate voltage drop = $I_1 R_{1e} \cos \Phi + I_1 X_{1e} \sin \Phi$

If the load has leading p.f. then we get the phasor diagram as shown in the Fig. 3. The I_2 leads V_2 by angle Φ_2 .

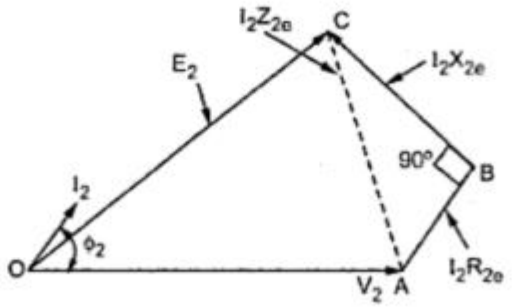


Fig. 3

In this case, the expression for approximate voltage drop remains same but the sign of $I_2 X_{2e}$ sin Φ reverses.

Approximate voltage drop = $I_2 R_{2e}$ cos Φ - $I_2 X_{2e}$ sin Φ .... Using referred to secondary values

= $I_1 R_{1e}$ cos Φ - $I_1 X_{1e}$ sin Φ ....Using referred to primary values

It can be noticed that for leading power factor $E_2 < V_2$.

For the unity power factor, the phasor diagram is simple and is shown in the Fig. 4. For this case, as cos Φ = 1 and sin Φ = 0, the approximate voltage drop is $I_2 R_{2e}$ or I_1R_{1e}.

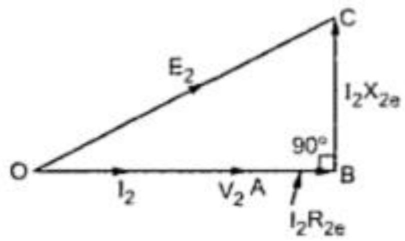


Fig. 4

Thus the general expression for the total approximate voltage drop is,

Approximate voltage drop = E_2 - V_2

= $I_{2e} R_{2e}$ cosΦ $I_{2e} X_{2e}$ sin Φ ....Using referred to secondary values

= $I_{1e} R_{1e}$ cos Φ $I_{1e} X_{1e}$ sin Φ ....Using referred to primary values

\+ sing for lagging power factor while - sign for leading power factor loads.

VOLTAGE REGULATION OF TRANSFORMER

Because of the voltage drop across the primary and secondary impedances it is observed that the secondary terminal voltage drops from its no load value (E_2) to load value (V_2) as load and load current increases.

This decrease in the secondary terminal voltage expressed as a fraction of the no load secondary terminal voltage is called regulation of a transformer.

The regulation is defined as change in the magnitude of the secondary terminal voltage, when full load i.e. rated load of specified power factor supplied at rated voltage is reduced to no load, with primary voltage maintained constant expressed as the percentage of the rated terminal voltage.

Let E_2 = Secondary terminal voltage on no load

V_2 = Secondary terminal voltage on given load

then mathematically voltage regulation at given load can be expressed as,

$$\% \text{ voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100$$

The ratio (E_2 - V_2 / V_2) is called per unit regulation.

The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of the power factor of the load. If V_2 is determined for full load and specified power factor condition the regulation is called full load regulation.

As load current increases, the voltage drops tend to increase V_2 and drops more and more. In case of lagging power factor $V_2 < E_2$ and we get positive voltage regulation, while for leading power factor $E_2 < V_2$ and we get negative voltage regulation.

The voltage drop should be as small as possible hence less the regulation better is the performance of a transformer.

<u>Expression for Voltage Regulation</u>

The voltage regulation is defined as,

$\%R = (E_2 - V_2 /V_2) \times 100 = (\text{Total voltage drop}/V_2) \times 100$

The expression for the total approximate voltage drop is already derived.

Total voltage drop = $I_2 R_{2e} \cos \Phi \pm I_2 X_{2e} \sin \Phi$

Hence the regulation can be expressed as,

$$\% R = \frac{I_2 R_{2e} \cos \phi \pm I_2 X_{2e} \sin \phi}{V_2} \times 100$$

'+' sing for lagging power factor while '-' sing for leading power factor loads.

The regulation van be further expressed interms of I_1 , V_1, R_{1e} and X_{1e}.

$V_2 /V_1 = I_1 /I_2 = K$

$\therefore \quad V_2 = KV_1 \quad , \quad I_2 = I_1/K$

while $\quad R_{1e} = R_{2e}/K^2$, $X_{1e} = X_{2e} /K^2$

Substituting in the regulation expression we get,

$$\% R = \frac{I_1 R_{1e} \cos \phi \pm I_1 X_{1e} \sin \phi}{V_1} \times 100$$

<u>Zero Voltage Regulation</u>

We have seen that for lagging power factor and unity power factor condition $V_2 < E_2$ and we get positive regulation. But as load becomes capacitive, V_2 starts increasing as load increase. At a certain leading power factor we get $E_2 = V_2$ and the regulation becomes zero. If the load is increased further, E_2 becomes less than V_2 and we get negative regulation.

$\therefore$ for zero voltage regulation,

$E_2 = V_2$

$\therefore$ $E_2 - V_2 = 0$

or $V_R \cos \Phi - V_x \sin \Phi = 0$ .... -ve sing as leading power factor

where $V_R = I_2 R_{2e} / V_2 = I_1 R_{1e} / V_1$ and $V_x = I_2 X_{2e} / V_2 = I_1 X_{1e} / V_1$

$\therefore$ $V_R \cos \Phi = V_x \sin \Phi$

$\therefore$ $\tan \Phi = V_R / V_x$

$\therefore$ $\cos \Phi = \cos \{\tan^{-1}(V_R / Vx)\}$

This is the leading p.f. at which voltage regulation becomes zero while supplying the load.

Constants of a Transformer

From the regulation expression we can define constants of a transformer.

$\%R = ((I_2 R_{2e} \cos \Phi \pm I_2 X_{2e} \sin \Phi) / E_2) \times 100$

$= \{(I_2 R_{2e} / E_2) \cos \Phi \pm (I_2 X_{2e} / E_2) \sin \Phi\} \times 100$

The ratio $(I_2 R_{2e} / E_2)$ or $(I_1 R_{1e} / E_1)$ is called per unit resistive drop and denoted as V_R.

The ratio $(I_2 X_{2e} / E_2)$ or $(I_1 X_{1e} / E_1)$ is called per unit reactive drop and is denoted as Vx.

The terms V_R and Vx are called constants of a transformer because for the rated output I_2, E_2, R_{1e}, X_{1e}, R_{2e}, X_{2e} are constants. The regulation can be expressed interms of V_R and Vx as,

$\%R = (V_R \cos \Phi \pm Vx \sin \Phi) \times 100$

On load condition, $E_2 = V_2$ and $E_1 = V_1$

where V_1 and V_2 are the given voltage ratings of a transformer. Hence V_R and Vx can be expressed as,

$V_R = I_2 R_{2e} / V_2 = I_1 R_{1e} / V_1$

and

$Vx = I_2 R_{2e} / V_2 = I_1 X_{1e} / V_1$

where V_1 and V_2 are no load primary and secondary voltages,

V_R and Vx can be expressed on percentage basis as,

Percentage resistive drop = $V_R \times 100$

Percentage reactive drop = $Vx \times 100$

Key Point : Note that and are also called per unit resistance and reactance respectively.

Losses in a Transformer

In a transformer, there exists two types of losses.

i) The core gets subjected to an alternating flux, causing core losses.

ii) The windings carry currents when transformer is loaded, causing copper losses.

1.1 Core or Iron Losses

Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

It is given by, hysteresis loss = $K_h B_m^{1.67} f v$ watts

where K_h = Hysteresis constant depends on material.

B_m = Maximum flux density.

f = Frequency.

v = Volume of the core.

The induced e.m.f. in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses. The eddy current loss is given by,

Eddy current loss = $K_e B_m^2 f^2 t^2$ watts/ unit volume

where K_e = Eddy current constant

t = Thickness of the core

As seen earlier, the flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant. Hence the flux density B_m in the core and hence both hysteresis and eddy current losses are constants at all the loads. Hence the core or iron losses are also called constant losses. The iron losses are denoted as P_i.

The iron losses are minimized by using high grade core material like silicon steel having very low hysteresis loop by manufacturing the core in the form of laminations.

1.2 Copper Losses

The copper losses are due to the power wasted in the form of $I^2 R$ loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

Total Cu loss = $I_1^2 R_1 + I_2^2 R_2 = I_1^2 (R_1 + R_2')= I_2^2 (R_2 +R_1')$

$= I_1^2 R_{1e} = I_2^2 R_{2e}$

The copper looses are denoted as. If the current through the windings is full load current, we get copper losses at full load. If the load on transformer is half then we get copper losses at half load which are less than full load copper losses. Thus copper losses are called variable losses. For transformer VA rating is or. As is constant, we can say that copper losses are proportional to the square of the KVA rating.

So, $P_{cu} \propto I^2 \propto (KVA)^2$

Thus for a transformer,

Total losses = Iron losses + Copper losses

$= P_i + P_{cu}$

Key point : It is seen that the iron losses depend on the supply voltage while the copper losses depend on the current. The losses are not dependent on the phase angle between voltage and current. Hence the rating of the transformer is expressed as a product of voltage and current and called VA rating of transformer. It is not expressed in watts or kilo watts. Most of the times, rating is expressed in KVA.

Losses: Additional Study:

Transformer losses are divided into losses in the windings, termed copper loss, and those in the magnetic circuit, termed iron loss. Losses in the transformer arise from:

Winding resistance

Current flowing through the windings causes resistive heating of the conductors. At higher frequencies, skin effect and proximity effect create additional winding resistance and losses.

Hysteresis losses

Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core. For a given core material, the loss is proportional to the frequency, and is a function of the peak flux density to which it is subjected.[42]

Eddy currents

Ferromagnetic materials are also good conductors and a core made from such a material also constitutes a single short-circuited turn throughout its entire length. Eddy currents therefore circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material. The eddy current loss is a complex function of the square of supply frequency and inverse square of the material thickness.[42] Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies use laminated or similar cores.

Magnetostriction

Magnetic flux in a ferromagnetic material, such as the core, causes it to physically expand and contract slightly with each cycle of the magnetic field, an effect known as magnetostriction. This produces the buzzing sound commonly associated with transformers[29] that can cause losses due to frictional heating. This buzzing is particularly familiar from low-frequency (50 Hz or 60 Hz) mains hum, and high-frequency (15,734 Hz (NTSC) or 15,625 Hz (PAL)) CRT noise.

Mechanical losses

In addition to magnetostriction, the alternating magnetic field causes fluctuating forces between the primary and secondary windings. These incite vibrations within nearby metalwork, adding to the buzzing noise and consuming a small amount of power.[43]

Stray losses

Leakage inductance is by itself largely lossless, since energy supplied to its magnetic fields is returned to the supply with the next half-cycle. However, any leakage flux that intercepts nearby conductive materials such as the transformer's support structure will give rise to eddy currents and be converted to heat.[44] There are also radiative losses due to the oscillating magnetic field but these are usually small.

EFFICIENCY OF A TRANSFORMER

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

∴ Power output = Power input - Total losses

∴ Power input = Power output + Total losses

= Power output + P_i + P_{cu}

The efficiency of any device is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expresses as,

η = Power output/power input

∴ η = Power output/(power output + P_i + P_{cu})

Now power output = $V_2 I_2 \cos \Phi$

where $\cos \Phi$ = Load power factor

The transformer supplies full load of current I_2 and with terminal voltage V_2.

P_{cu} = Copper losses on full load = $I_2^2 R_{2e}$

∴ $\eta = (V_2 I_2 \cos \Phi_2)/(V_2 I_2 \cos \Phi_2 + P_i + I_2^2 R_{2e})$

But $V_2 I_2$ = VA rating of a transformer

∴ η = (VA rating x $\cos \Phi$) / (VA rating x $\cos \Phi$ + P_i + $I_2^2 R_{2e}$)

$$\therefore \quad \% \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}} \times 100$$

This is full load percentage efficiency with,

I_2 = Full load secondary current

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

Let n =Fraction by which load is less than full load = Actual load/Full load
For example, if transformer is subjected to half load then,
n = Half load/Full load = (1/2)/2 = 0.5

when load changes, the load current changes by same proportion.

∴ new $I_2 = n (I_2)$ F.L.

Similarly the output $V_2 I_2 \cos\Phi_2$ also reduces by the same fraction. Thus fraction of VA rating is available at the output.

Similarly as copper losses are proportional to square of current then,

new $P_{cu} = n^2 (P_{cu})$ F.L.

Key Point : So copper losses get reduced by n^2.

In general for fractional load the efficiency is given by,

$$\% \eta = \frac{n\,(\text{VA rating})\cos\phi}{n\,(\text{VA rating})\cos\phi + P_i + n^2\,(P_{cu})\,\text{F.L.}} \times 100$$

where n = Fraction by which load power factor lagging, leading and unity the efficiency expression does not change, and remains same.

O.C. AND S.C. TESTS ON SINGLE PHASE TRANSFORMER

The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method. In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,

1. Open circuit test (O.C Test)
2. Short circuit test (S.C.Test)

The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer. The advantage of this method is that without much power loss the tests can be performed and results can be obtained. Let us discuss in detail how to perform these tests and how to use the results to calculate equivalent circuit parameters.

1.1 Open Circuit Test (O.C. Test)

The experimental circuit to conduct O.C test is shown in the Fig. 1.

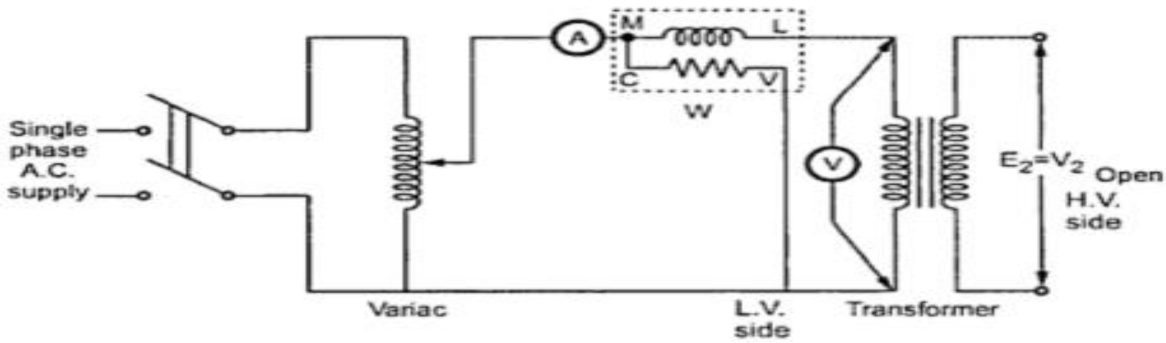


Fig 1. Experimental circuit for O.C. test

The transformer primary is connected to a.c. supply through ammeter, wattmeter and variac. The secondary of transformer is kept open. Usually low voltage side is used as primary and high voltage side as secondary to conduct O.C test.

The primary is excited by rated voltage, which is adjusted precisely with the help of a variac. The wattmeter measures input power. The ammeter measures input current. The voltemeter gives the value of rated primary voltage applied at rated frequency.

Sometimes a voltmeter may be connected across secondary to measure secondary voltage which is $V_2 = E_2$ when primary is supplied with rated voltage. As voltmeter resistance is very high, though voltmeter is connected, secondary is treated to be open circuit as voltmeter current is always negligibly small.

When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded.

The observation table is as follows

V_o volts	I_o amperes	W_o watts
Rated		

V_o = Rated voltage
W_o = Input power
I_o = Input current = no load current

As transformer secondary is open, it is on no load. So current drawn by the primary is no load current I_o. The two components of this no load current are,

$I_m = I_o \sin \Phi_o$
$I_c = I_o \cos \Phi_o$
where $\cos \Phi_o$ = No load power factor
And hence power input can be written as,
$W_o = V_o I_o \cos \Phi_o$
The phasor diagram is shown in the Fig. 2.

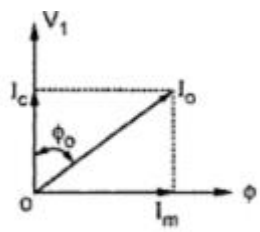


Fig. 2

As secondary is open, $I_2 = 0$. Thus its reflected current on primary is also zero. So we have primary current $I_1 = I_o$. The transformer no load current is always very small, hardly 2 to 4 % of its full load value. As $I_2 = 0$, secondary copper losses are zero. And $I_1 = I_o$ is very low hence copper losses on primary are also very very low. Thus the total copper losses in O.C. test are negligibly small. As against this the input voltage is rated at rated frequency hence flux density in the core is at its maximum value. Hence iron losses are at rated voltage. As output power is zero and copper losses are very low, the total input power is used to supply iron losses. This power is measured by the wattmeter i.e. W_o. Hence the wattmeter in O.C. test gives iron losses which remain constant for all the loads.

∴ $W_o = P_i$ = Iron losses

Calculations : We know that,

$W_o = V_o I_o \cos \Phi$

$\cos \Phi_o = W_o/(V_o I_o)$ = no load power factor

Once $\cos \Phi_o$ is known we can obtain,

$I_c = I_o \cos \Phi_o$

and $I_m = I_o \sin \Phi_o$

Once I_c and I_m are known we can determine exciting circuit parameters as,

$R_o = V_o/I_c \ \Omega$

and $X_o = V_o/I_m \ \Omega$

Key Point : The no load power factor $\cos \Phi_o$ is very low hence wattmeter used must be low power factor type otherwise there might be error in the results. If the meters are connected on secondary and primary is kept open then from O.C. test we get R_o' and X_o' with which we can obtain R_o and X_o knowing the transformation ratio K.

1.2 Short Circuit Test (S.C. Test)

In this test, primary is connected to a.c. supply through variac, ammeter and voltmeter as shown in the Fig. 3.

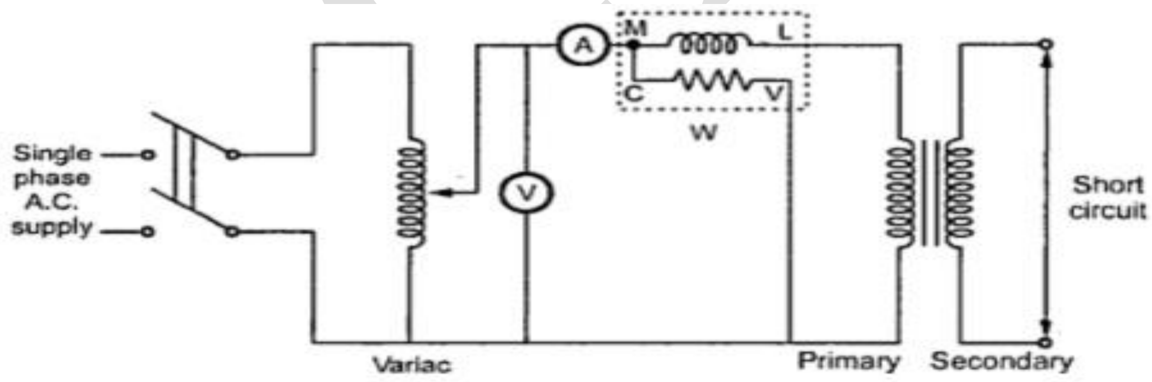


Fig. 3 Fig 1. Experimental circuit for O.C. test

The secondary is short circuited with the help of thick copper wire or solid link. As high voltage side is always low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side.

As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current. Such large current can cause overheating and burning of the transformer. To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter. The low voltage can be adjusted with the help of variac. Hence this test is also called low voltage test or reduced voltage test. The wattmeter reading as well as voltmeter, ammeter readings are recorded. The observation table is as follows,

V_{sc} volts	I_{sc} amperes	W_{sc} watts
	Rated	

Now the current flowing through the windings are rated current hence the total copper loss is full load copper loss. Now the voltage supplied is low which is a small fraction of the rated voltage. The iron losses are function of applied voltage. So the iron losses in reduced voltage test are very small. Hence the wattmeter reading is the power loss which is equal to full load copper losses as iron losses are very low.

$\therefore$ $W_{sc} = (P_{cu})$ F.L. = Full load copper loss

Calculations : From S.C. test readings we can write,

$W_{sc} = V_{sc} I_{sc} \cos \Phi_{sc}$

$\therefore$ $\cos \Phi sc = V_{sc} I_{sc} / W_{sc}$ = short circuit power factor

$W_{sc} = I_{sc}^2 R_{1e}$ = copper loss

$\therefore$ $R_{1e} = W_{sc} / I_{sc}^2$

while $Z_{1e} = V_{sc} / I_{sc} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$

$\therefore$ $X_{1e} = \sqrt{(Z_{1e}^2 - R_{1e}^2)}$

Thus we get the equivalent circuit parameters R_{1e}, X_{1e} and Z_{1e}. Knowing the transformation ratio K, the equivalent circuit parameters referred to secondary also can be obtained.

Important Note : If the transformer is step up transformer, its primary is L.V. while secondary is H.V. winding. In S.C. test, supply is given to H.V. winding and L.V is shorted. In such case we connect meters on H.V. side which is transformer secondary through for S.C. test purpose H.V side acts as primary. In such case the parameters calculated from S.C. test readings are referred to secondary which are R_{2e}, Z_{2e} and X_{2e}. So before doing calculations it is necessary to find out where the readings are recorded on transformer primary or secondary and accordingly the parameters are to be determined. In step down transformer, primary is high voltage itself to which supply is given in S.C. test. So in such case test results give us parameters referred to primary i.e. R_{1e}, Z_{1e} and X_{1e}.

Key point : In short, if meters are connected to primary of transformer in S.C. test, calculations give us R_{1e} and Z_{1e} if meters are connected to secondary of transformer in S.C. test calculations give us R_{2e} and Z_{2e}.

1.3 Calculation of Efficiency from O.C. and S.C. Tests

We know that,

From O.C. test, $W_o = P_i$

From S.C. test, $W_{sc} = (P_{cu})$ F.L.

$$\therefore \quad \% \eta \text{ on full load } = \frac{V_2 (I_2) \text{F.L.} \cos\phi}{V_2 (I_2) \text{F.L.} \cos + W_o + W_{sc}} \times 100$$

Thus for any p.f. cos Φ_2 the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos\phi}{n \times (\text{VA rating}) \times \cos\phi + W_o + n^2 W_{sc}} \times 100$$

where n = fraction of full load

or

$$\% \eta = \frac{n V_2 I_2 \cos\phi}{n V_2 I_2 \cos\phi + W_o + n^2 W_{sc}} \times 100$$

where $I_2 = n (I_2)$ F.L.

1.4 Calculation of Regulation

From S.C. test we get the equivalent circuit parameters referred to primary or secondary.

The rated voltages V_1, V_2 and rated currents (I_1) F.L. and (I_2) F.L. are known for the given transformer. Hence the regulation can be determined as,

$$\% R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100$$

$$= \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100$$

where I_1, I_2 are rated currents for full load regulation.

For any other load the currents I_1, I_2 must be changed by fraction n.

$\therefore$ I_1, I_2 at any other load = n (I_1) F.L., n (I_2) F.L.

Key Point : Thus regulation at any load and any power factor can be predetermined, without actually loading the transformer.

Example 1 : A 5 KVA, 500/250 V, 50 Hz, single phase transformer gave the following readings,
O.C. Test : 500 V, 1 A, 50 W (L.V. side open)
S.C. Test : 25 V, 10 A, 60 W (L.V. side shorted)
Determine : i) The efficiency on full load, 0.8 lagging p.f.
ii) The voltage regulation on full load, 0.8 leading p.f.
iii) The efficiency on 60% of full load, 0.8 leading p.f.
iv) Draw the equivalent circuit referred to primary and insert all the values in it.

Solution : In both the tests, meters are on H.V. side which is primary of the transformer. Hence the parameters obtained from test results will be referred to primary.

From O.C. test, $V_o = 500$ V, $I_o = 1$ A, $W_o = 50$ W

∴ $\cos \Phi_o = W_o/V_o I_o = 50/(500 \times 1) = 0.1$

∴ $I_c = I_o \cos = 1 \times 0.1 = 0.1$ A

and $I_m = I_o \sin \Phi_o = 1 \times 0.9949 = 0.9949$ A

∴ $R_o = V_o / I_c = 500/0.1 = 5000\ \Omega$

and $X_o = V_o/I_m = 500/0.9949 = 502.52\ \Omega$

and $W_o = P_i =$ iron losses = 50 W

From S.C. test, $V_{sc} = 25$ V, $I_{sc} = 10$ A, $W_{sc} = 60$ W

∴ $R_{1e} = W_{sc}/I_{sc}^2 = 60/(10)^2 = 0.6\ \Omega$

$Z_{1e} = V_{sc}/I_{sc} = 25/10 = 2.5\ \Omega$

∴ $X_{1e} = \sqrt{(2.5^2 - 0.6^2)} = 2.4269\ \Omega$

(I_1) F.L. = VA rating/V_1

$= (5 \times 10^3)/500 = 10$ A

and $I_{sc} = (I_1)$ F.L.

∴ $W_{sc} = (P_{cu})$ F.L. = 60 W

i) η on full load, cos = 0.8 lagging

$$\% \eta = \frac{(\text{VA rating}) \cos \phi_2}{(\text{VA rating}) \cos \phi_2 + P_i + (P_{cu}) \text{ F. L.}} \times 100$$

$$= \frac{5 \times 10^3 \times 0.8}{5 \times 10^3 \times 0.8 + 50 + 60} \times 100 = \mathbf{97.32\ \%}$$

ii) Regulation on full load, $\cos \Phi_2 = 0.8$ leading

$$\% R = \frac{(I_1) \text{ F. L. } R_{1e} \cos \phi - (I_1) \text{ F. L. } X_{1e} \sin \phi}{V_1} \times 100$$

$$= \frac{10 \times 0.6 \times 0.8 - 10 \times 2.4269 \times 0.6}{500} \times 100$$

$= -1.95\ \%$

iii) For 60% of full load, n = 0.6 and $\cos \Phi_2 = 0.8$ leading]

∴ P_{cu} = copper loss on new load = $n^2 \times (P_{cu})$ F.L.

$= (0.6)^2 \times 60 = 21.6$ W

$= 97.103\ \%$

iv) The equivalent circuit referred to primary is shown in the Fig. 4.

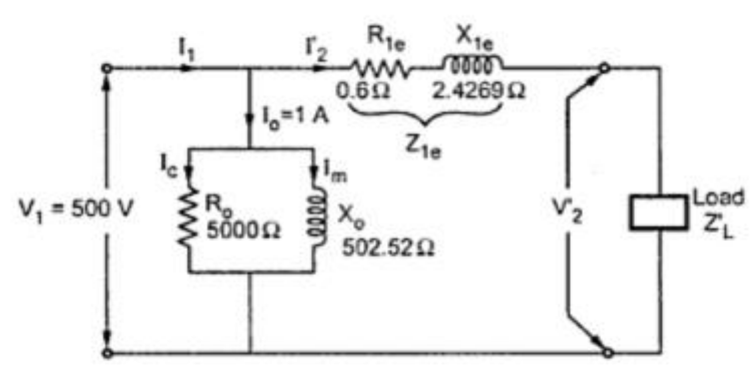


Fig. 4

Example 2 : The open circuit and short circuit tests on a 10 KVA, 125/250 V, 50 Hz, single phase transformer gave the following results :

O.C. test : 125 V, 0.6 A, 50 W (on L.V. side)
S.C. test : 15 V, 30 A. 100 W (on H.V. side)
Calculate : i) copper loss on full load
ii) full load efficiency at 0.8 leading p.f.
iii) half load efficiency at 0.8 leading p.f.
iv) regulation at full load, 0.9 leading p.f.
Solution : From O.C. test we can weite,

$W_o = P_i = 50$ W = Iron loss

From S.C. test we can find the parameters of equivalent circuit. Now S.C. test is conducted on H.V. side i.e. meters are on H.V. side which is transformer secondary. Hence parameters from S.C. test results will be referred to secondary.

$V_{sc} = 15$ V, $I_{sc} = 30$ A, $W_{sc} = 100$ W

∴ $R_{2e} = W_{sc}/(I_{sc})^2 = 10/(30)^2 = 0.111\Omega$

$Z_{1e} = V_{sc}/I_{sc} = 15/30 = 0.5\ \Omega$

∴ $X_{2e} = \sqrt{(Z_{2e}^2 - R_{2e}^2)} = 0.4875\ \Omega$

i) Copper loss on full load

(I_2) F.L. = VA rating/V_2 = $(10 \times 10^3)/250 = 40$ A

In short circuit test, $I_{sc} = 30$ A and not equal to full load value 40 A.

Hence W_{sc} does not give copper loss on full load

∴ $W_{sc} = P_{cu}$ at 30 A = 100 W

Now $P_{cu} \propto I^2$

$(P_{cu}$ at 30 A$)/(P_{cu}$ at 40 A$) = (30/40)^2$

$100/(P_{cu}$ at 40 A$) = 900/1600$

P_{cu} at 40 A = 177.78 W

∴ (P_{cu}) F.L. = 177.78 W

ii) Full load η , cos $\Phi_2 = 0.8$

$$\%\ \eta \text{ on full load} = \frac{V_2(I_2)\text{ F. L. }\cos\phi_2}{V_2(I_2)\text{ F. L.}\cos\phi_2 + P_i + (P_{cu})\text{ F. L.}} \times 100$$

$$= \frac{250\times40\times0.8}{250\times40\times0.8+50+177.78}\times100 = \mathbf{97.23\ \%}$$

iii) Half load η , cos $\Phi_2 = 0.8$

n = 0.5 as half load, (I_2) H.L. = 0.5 x 40 =

$$\therefore \quad \%\,\eta \text{ on half load} = \frac{V_2(I_2)\text{ H. L. }\cos\phi_2}{V_2(I_2)\text{ H. L.}\cos\phi_2 + P_i + n^2(P_{cu})\text{ F. L.}} \times 100$$

$$= \frac{n\,(\text{VA rating})\cos\phi_2}{n\,(\text{VA rating})\cos\phi_2 + P_i + n^2(P_{cu})\text{ F. L.}} \times 100$$

$$= \frac{0.5\times10\times10^3\times0.8}{0.5\times10\times10^3\times0.8+50+(0.5)^2\times177.78} \times 100$$

= 97.69%

iv) Regulation at full load, cos Φ = 0.9 leading

$$\%\,R = \frac{(I_2)\text{ F.L.}R_{2e}\cos\phi-(I_2)\text{ F.L. }X_{2e}\sin\phi}{V_2} \times 100$$

$$= \frac{40\times0.111\times0.9-40\times0.4875\times0.4358}{250} \times 100$$

= -1.8015%

15.Additional topics

1)AC transients
2)Auto transformers

16 University previous Question papers

R09

Code No: 54019

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech II Year II Semester Examinations, May - 2015

PRINCIPLES OF ELECTRICAL ENGINEERING

(Common to ECE, ETM)

Time: 3 hours Max. Marks: 75

Answer any five questions

All questions carry equal marks

- - -

1.a) Explain the procedure to find the solution of a differential equation using Laplace transform method.

b) Find i(t) in the circuit in figure 1 for t > 0. Assume that the switch has been closed for a long time.

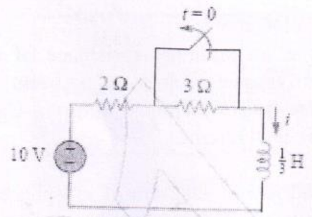


Figure: 1

[7+8]

2.a) Determine the y parameters for the two-port shown in figure 2.

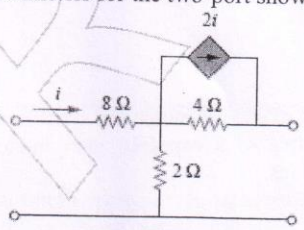


Figure: 2

b) Obtain the condition of transmission parameters for two network connected in cascade. [8+7]

3.a) Explain the variations of characteristic impedance (Z_0), attenuation constant (α) and phase constant (β) with frequency with the help of neat sketch in band elimination filter.

b) Design a composite high pass filter to operate into a load of 800Ω and have a cut off frequency of 1.2 kHz. The filter is to have one constant k section, one m-derived section with $f_\infty = 1.1$ kHz and suitable termination half section. [8+7]

4.a) Design symmetrical lattice attenuator with 30dB attenuation, working into 600Ω impedance. N = Antilog$_{10}$(D/20) = Antilog$_{10}\left[\frac{20}{20}\right]$ 10.

b) Derive the design equations for
i) Symmetrical T attenuator.
ii) Symmetrical π attenuator. [7+8]

5.a) Explain applications of various d.c. generators.
b) A 500V dc generator is supplying a 30kW load has a resistance of 0.4Ω, shunt field resistance of 300Ω. Determine the armature current, induced emf. Allow a contact drop is 1V per brush. [7+8]

6.a) Explain the various losses in a DC motor.
b) A 250V d.c. shunt motor has an armature resistance of 0.5Ω and shunt field resistance of 300Ω, when driving at 600rpm at constant load. Armature takes 20A, speed is required to rise from 600rpm to 800rpm. Calculate the additional resistance to be inserted in the field circuit. [7+8]

7.a) Draw and explain the no-load phasor diagram of a 1-phase transformer. Discuss how primary leakage flux is accounted for in the phasor diagram.
b) A transformer when tested on full load is found to have copper loss of 1.8% and reactance drop of 4.8%. Calculate the full load regulation at power factors of
i) 0.8 lag
ii) UPF
iii) 0.707 leading. [8+7]

8.a) Discuss various applications of stepper motor and synchros.
b) Explain the working principle of a capacitor start induction motor. And draw the speed-torque characteristics. [8+7]

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R09

Code No: 09A40401

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year II Semester Examinations, June-2014

PRINCIPLES OF ELECTRICAL ENGINEERING

(Common to ECE, ETM)

Time: 3 Hours **Max. Marks: 75**

Answer any Five Questions

All Questions Carry Equal Marks

\- - -

.a) A capacitor with initial voltage v_o is connected to resistor of RΩ at $t = 0$, derive the expression for the voltage across the capacitor and current through the capacitor at any time $t > 0$ and plot the waveforms.

b) Find $i\,(0^+)$ in the circuit shown in Figure 1.

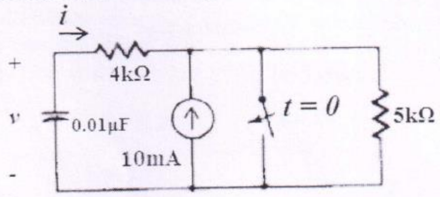

Figure: 1

Find ABCD parameters of the circuit in Figure 2.

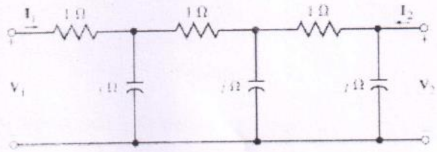

Figure: 2

A low pass π-section filter consists of an inductance of 25 mH in series arm and two capacitors of 0.2μF in shunt arms. Calculate the cut-off frequency, design impedance, attenuation at 5 KHz and phase shift at 2 KHz. Also find the characteristic impedance at 2 KHz.

ı) Explain T-type attenuator.

›) Design a T-type attenuator to give an attenuation of 60dB and to work in a line of 500Ω impedance.

ı) Derive an expression for the induced emf in the armature of a DC Machine.

›) A 4-pole, lap-wound, dc shunt generator has a useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns each of 0.004 ohms resistance. Calculate the terminal voltage when running at 900 rpm, if the armature current is 50A.

) Discuss various methods of speed control of dc shunt motor?

) A 250V DC shunt motor takes 4A when running unloaded. Its armature and field resistances are 0.3 Ω and 250 Ω respectively. Calculate the efficiency when the dc shunt motor taking a current of 60A.

7.a) Draw the phasor diagram of a single phase transformer under load conditions for lagging power factor.

b) 6600/400V, 50 Hz, single phase Transformer has a net cross-sectional area of the core of 428 cm^2. The maximum flux density in the core is 1.5 Tesla. Calculate the number of turns in the primary and secondary windings.

8. Write short notes on the following:

a) Capacitor motors.

b) Shaded pole motor.

c) Stepper motor.

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R09

Code No: 09A40401

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD
B.Tech II Year II Semester Examinations, November / December-2013
PRINCIPLES OF ELECTRICAL ENGINEERING
(Common to ECE, ETM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks
\- - -

1.a) Obtain the behaviour and characteristics of an R-L Circuit subjected to unit step excitation and derive the expression of transient current flows in the circuit.

b) Find i(t) in the network shown below in figure 1, when the switch K is closed at $t=0^+$. A current of 2A was flowing at $t=0^-$ in the inductor. [15]

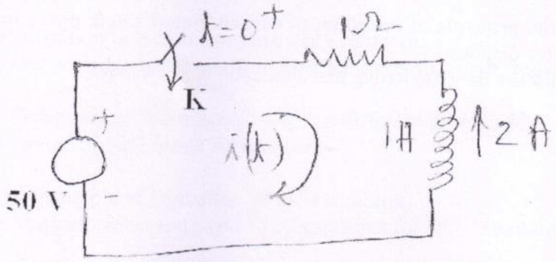

figure 1

2.a) For a passive two port network derive the expression for transmission and hybrid parameters.

b) Find the ABCD parameters of the network shown below in figure 2. [15]

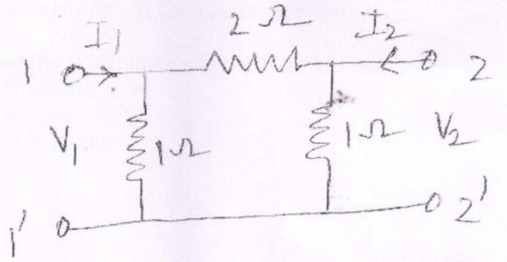

figure 2

3.a) Explain the nature of characteristic impedance, phase angle, cross over frequency in pass and stop bands.

b) Derive the important relations in constant-k low pass filter and constant-k high pass filter. [15]

4. Discuss the functioning and significance of T-type and π-type attenuators. [15]

5.a) Derive the EMF equation of D.C generator.

b) A 110 V d.c shunt generator delivers a load current of 50 A. The armature resistance is 0.2 Ω, and the field resistance is 55 Ω. The generator, rotating at a speed of 1800 rpm, has 6 poles, lap-wound, and a total of 360 conductors. Calculate no-load voltage at the armature and the flux per pole. [15]

6. Draw and explain the D.C series-wound, shunt-wound and compound wound motor characteristics and their applications. [15]

7.a) Describe the principle of operation and constructional features of a single phase transformer.

b) Explain i) Efficiency ii) Voltage regulation of a 1-phase transformer. [15]

8.a) Explain the principle of operation of capacitor-start single phase induction motor with phasor diagram.

b) Briefly discuss the functioning and applications of synchros. [15]

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R09

Code No: R09220401

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B. Tech II Year - II Semester Examinations, November/December, 2012

PRINCIPLES OF ELECTRICAL ENGINEERING

(COMMON TO ECE, ETM)

Time: 3 hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

\- - -

1.a) Why voltage across capacitor cannot change instantaneously? Explain.
b) Explain transient response in series R-L-C circuit with D.C excitation. [15]

2.a) Explain why Admittance Parameters are called as short circuit parameters.
b) Find Z-parameters for T network. [15]

3.a) Explain Realisation of active low pass filter in detail.
b) Design a second order Butterworth low pass filter with cutoff frequency of 1kHz. [15]

4.a) What are Symmetrical Attenuators? Explain different types.
b) Explain in detail about Lattice Attenuator. [15]

5.a) Explain the Principle of Operation of DC Generator.
b) Draw the Magnetization and Load Characteristics for DC Generators. [15]

6.a) Why DC series motor can never be operated on no-load condition. Explain.
b) What is speed control? Explain the Speed Control of DC Shunt Motor by using Armature Voltage Control method. [15]

7.a) What are the different types of Losses in a transformer. How they are minimized?
b) Explain clearly bout Short Circuit test on transformer. [15]

8.a) Explain working Principle of single phase induction motor.
b) Write short notes on
i) AC Servomotor
ii) Shaded Pole motors. [15]

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II B.TECH - II SEMESTER EXAMINATIONS, APRIL/MAY, 2011
PRINCIPLES OF ELECTRICAL ENGINEERING
(COMMON TO ELECTRICAL AND COMMUNICATION ENGINEERING & ELECTRONICS AND TELEMATICS ENGINEERING)
Time: 3hours Max. Marks: 75
Answer any FIVE questions
All Questions Carry Equal Marks
- - -

1.a) For the circuit shown below Figure. 1, find the current equation when switch S is opened at t = 0.

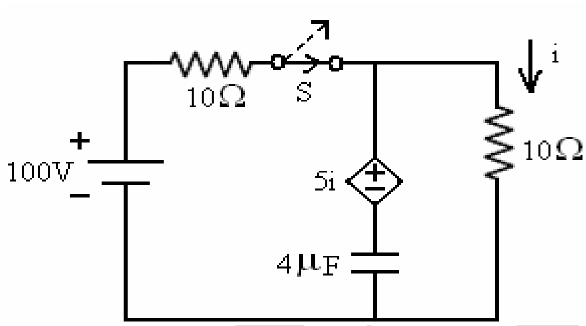


Figure. 1

b) Convert the current source shown below Figure. 2 in to a voltage source in the S domains. [7+8]

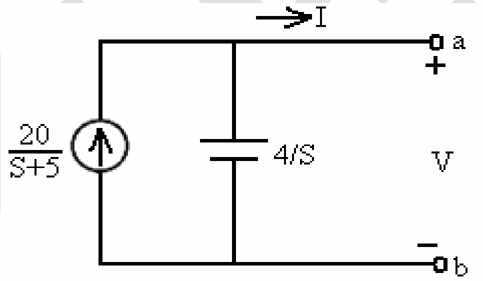


Figure. 2

2. Find Z and Y parameter of the network shown below Figure. 3. [15]

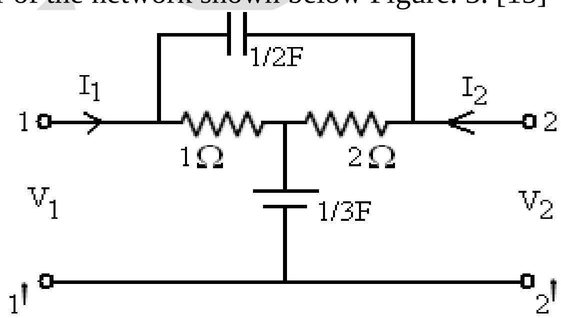


Figure. 3

3. Design a band elimination filter having a design impedance of 600Ω and cut – off frequencies f_1 = 2 KHz and f_2 = 6 KHZ. [15]

4. Explain T – type attenuator and also design a T – type attenuator to give an attenuation of 60dB and to work in a line of 500Ω impedance. [15]

5. What are the different types of dc generators? Show the connection diagrams and load characteristics of each type. [15]

6.a) Explain why a dc series motor should never run unloaded.

b) A 200V, 14.92kW, dc shunt motor when tested by Swinburne's method gave the following test results.

Running light: Armature current of 6.5 A and field current = 2.2A

With armature locked: I_a=70A when potential difference of 3V was applied to the brusher.

Estimate efficiency of motor when working under full load. [5+10]

7. A 50Hz, 1Ø, 100 KVA transformer has full load copper loss of 1200W and its iron loss is 960W. Calculate:

a) The efficiency at full load, unity power factor.

b) The efficiency at half load, 0.8 power factor.

c) The efficiency at 7.5% of full load, 0.7 power factor. [15]

8. Write short notes on the following:

a) AC Servo motors.

b) Shaded pole motor.

c) Synchros. [15]

II B.TECH - II SEMESTER EXAMINATIONS, APRIL/MAY, 2011
PRINCIPLES OF ELECTRICAL ENGINEERING
(COMMON TO ELECTRICAL AND COMMUNICATION ENGINEERING & ELECTRONICS AND TELEMATICS ENGINEERING)
Time: 3hours Max. Marks: 75
Answer any FIVE questions
All Questions Carry Equal Marks
- - -

1.a) For the below circuit (Figure.1), find the current in 20Ω when the switch is opened at t = 0.

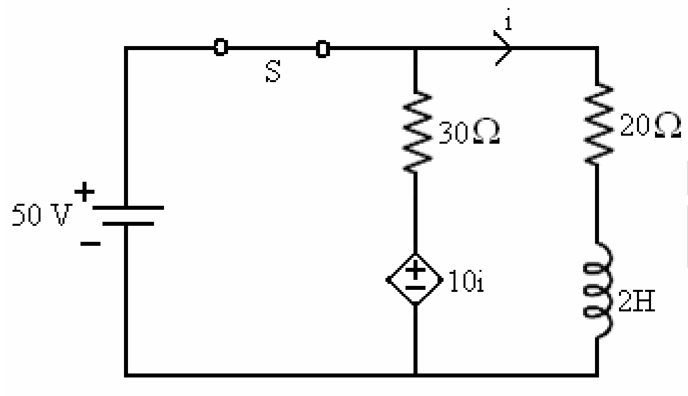


Figure. 1

b) Transform the below circuit (Figure. 2) in to 'S' domain and determine the Laplace transform impedance. [7+8]

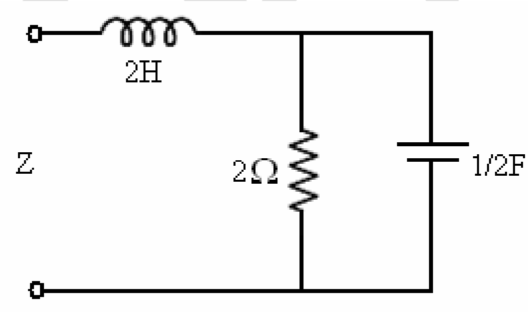


Figure. 2

2. Determine Y – parameters of the below (Figure. 3) network. Hence determine the h-parameters. [15]

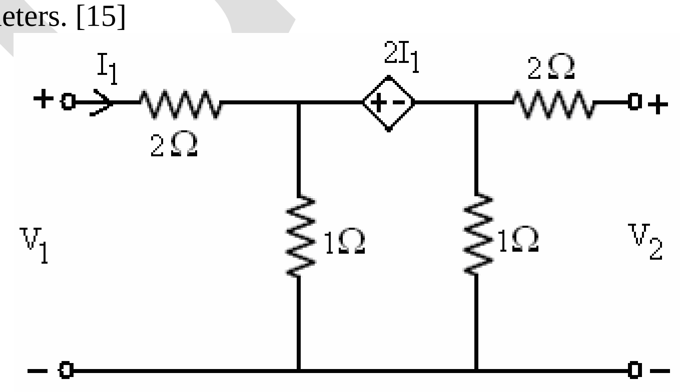


Figure. 3

3. Design a m – derived high pass filter with a cut – off frequency of 10KHz; design impedance of 5Ω and m = 0.4. [15]

4. Explain the lattice attenuator and also design a lattice attenuator to have a characteristic impedance of 800Ω and attenuation of 20 dB. [15]

5. State the principle of operation of a dc generator and derive the expression for the emf generated. [15]

6.a) Derive the torque equation of a dc motor.

b) A 4 pole, 500V dc shunt motor has 700 wave connected armature conductors. The full load armature current is 60 A and the flux per pole is 30mWb. Calculate the full load speed if the motor armature resistance is 0.2Ω and brush drop is 1V per brush. [7+8]

7. Draw the phasor diagram of a single phase transformer under load conditions for lagging, leading and unity power factors. [15]

8. Explain in detail the principle of operation and constructional details of a shaded pole motor. [15]

II B.TECH - II SEMESTER EXAMINATIONS, APRIL/MAY, 2011
PRINCIPLES OF ELECTRICAL ENGINEERING
(COMMON TO ELECTRICAL AND COMMUNICATION ENGINEERING & ELECTRONICS AND TELEMATICS ENGINEERING)
Time: 3hours Max. Marks: 75
Answer any FIVE questions
All Questions Carry Equal Marks
- - -

1.a) For the below circuit (Figure. 1), find the current equation i(t), when the switch is opened at t = 0.

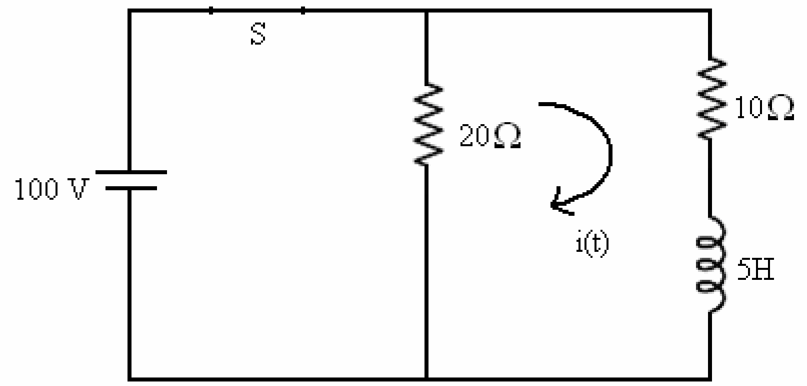


Figure. 1

b) Transform the below circuit (Figure.2) in to 'S' domain and determine the laplace impedance. [7+8]

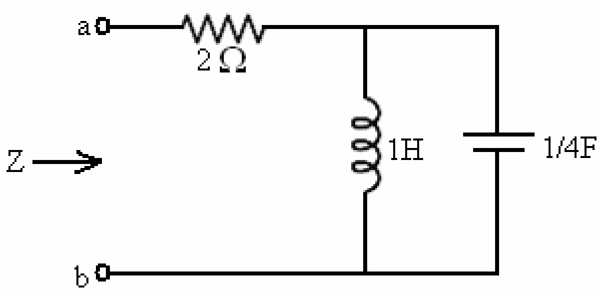


Figure. 2

2. Determine the transmission parameter and hence determine the short circuit admittance parameters for the below circuit (Figure.3). [15]

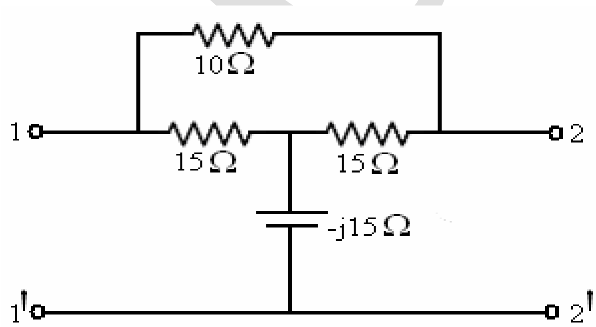


Figure. 3

3. What is a constant – K low pass filter, derive its characteristics impedance. [15]

4. Explain π – type attenuator and also design it to give 20db attenuation and to have characteristic impedance of 100Ω. [15]

5. Explain in detail the construction and principle of operations of DC generators. [15]

6. Discuss in detail the different methods of speed control of a dc motor. [15]

7. Open circuit and short circuit tests on a 5 KVA, 220/400V, 50 Hz, single phase transformer gave the following results:
OC Test: 220V, 2A, 100W (lv side)
SC Test: 40V, 11.4A, 200W (hv side)
Determine the efficiency and approximate regulation at full load, 0.9 power factor lagging. [15]

8. Write a short note on the following:
a) Capacitor motors.
b) Stepper motor.
c) AC tachometers. [15]

II B.TECH - II SEMESTER EXAMINATIONS, APRIL/MAY, 2011
PRINCIPLES OF ELECTRICAL ENGINEERING
(COMMON TO ELECTRICAL AND COMMUNICATION ENGINEERING & ELECTRONICS AND TELEMATICS ENGINEERING)
Time: 3hours Max. Marks: 75
Answer any FIVE questions
All Questions Carry Equal Marks
- - -

1.a) Determine the current i for t ≥ 0 if initial current i(0) = 1 for the below circuit (Figure. 1).

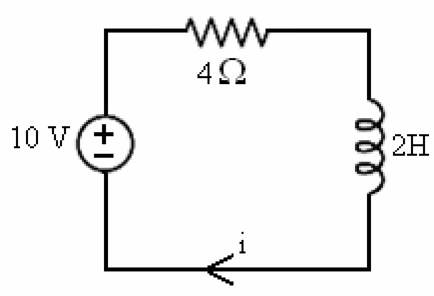


Figure. 1

b) Switch is opened at t = 0 in the below circuit (Figure. 2). Then find the current 'i'.

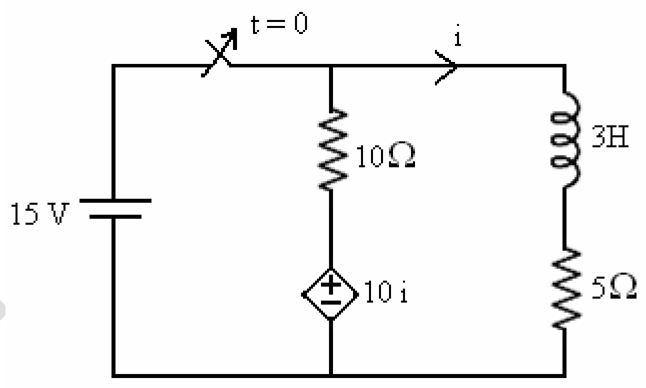
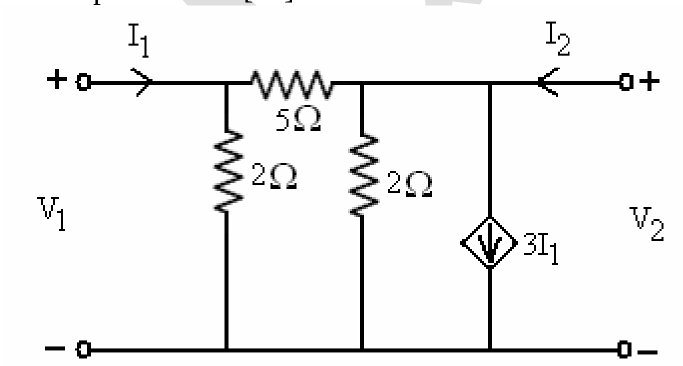


Figure. 2

2. Obtain Z parameters of the below circuit (Figure. 3) and from there Z – parameters derive h – parameters. [15]



3. A low pass π section filter consists of an inductance of 25 mH in series arm and two capacitors of 0.2μF in shunt arms. Calculate the cut – off frequency, design impedance, attenuation at 5 KHz and phase shift at 2 KHz. Also find the characteristic impedance at 2 KHz. [15]

4. Explain Bridged – T attenuator and also design it with an attenuation of 20 dB and terminated in a load of 500Ω. [15]

5. A 6 – pole dc shunt generator with a wave – wound armature has 960 conductors. It runs at a speed of 500 rpm. A load of 20Ω is connected to the generator at a terminal voltage of 240V. The armature and field resistances are 0.3Ω and 240Ω respectively. Find the armature current, the induced emf and flux per pole. [15]

6. Sketch the speed – load characteristics of a dc shunt, series and compound motors. Account for the shape of the above characteristic curves. [15]

7.a) Derive the expression for the induced emf of a transformer.
b) A 125 KVA transformer having primary voltage of 2000V at 50 Hz has 182 primary and 40 secondary turns. Neglecting losses, calculate:
i) The full load primary and secondary currents.
ii) The no-load secondary induced emf.
iii) Maximum flux in the core. [7+8]

8. Draw the circuit diagram of capacitor – start, capacitor – run single phase induction motor and explain its working. Where this type of motor is commonly used? [15]

B.Tech II Year - II Semester Examinations, December-2011 / January-2012

PRINCIPLES OF ELECTRICAL ENGINEERING

(COMMON TO ECE, ETM)

Time: 3 hours Max. Marks: 80

Answer any five questions

All questions carry equal marks

- - -

1.a) Find v(t) for t ≥ 0, when the switch is opened at t = 0 for the circuit shown in Fig.1.

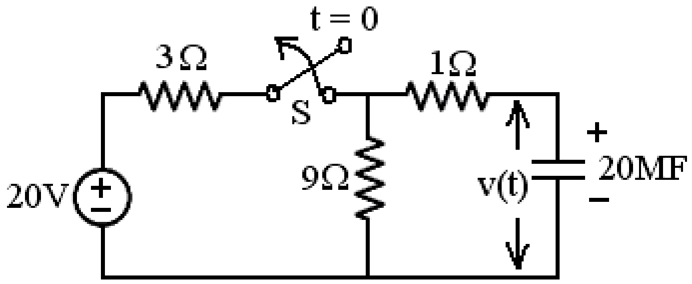


Fig.1

b) Determine i(t) for the circuit shown in Fig.2. When the switch is closed at t = 0. Assume initial current through inductor is zero. [8+7]

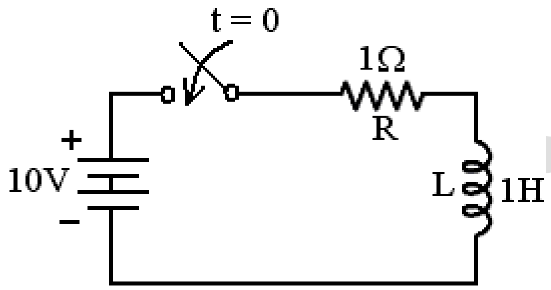


Fig.2

2. Determine the Y – parameters for the two – port network shown in Fig.3 and also find g – parameters. [15]

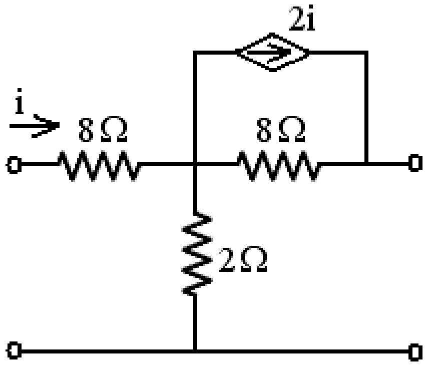


Fig.3

3. Design a band stop, constant – K filter with cut off frequencies of 4 KHz and 10 KHz and nominal characteristic impedance of 500 Ω. [15]

4.a) Explain about a symmetrical π – attenuator.

b) Design a symmetrical π – attenuator to provide attenuation of 20dB and design impedance of and design impedance of 400 Ω [7+8]

5.a) Derive an expression for the induced emf in the armature of a DC Machine.

b) The armature of a 4 – pole lap wound shunt generator has 480 conductors. The flux per pole is 0.05 Wb. The armature and field resistances are 0.05 Ω and 50 Ω. Find the speed of the machine when supplying 450A at a terminal voltage of 250V. [7+8]

6.a) Write about the various losses occurring in a dc motor and name the parts of the machine in which these occur.

b) A 250V DC shunt motor takes 4A when running unloaded. Its armature and field resistances are 0.3 Ω and 250 Ω respectively. Calculate the efficiency when the dc shunt motor taking a current of 60A. [7+8]

7.a) Explain the principle of operation of 1-Ø Transformer.

b) Derive the equivalent circuit of 1-Ø Transformer and discuss its significance.

[7+8]

8. Write short notes on the following:

a) AC Tachometers.

b) Stepper motors.

c) Capacitor motors. [15]

B.Tech II Year - II Semester Examinations, December-2011 / January-2012

PRINCIPLES OF ELECTRICAL ENGINEERING

(COMMON TO ECE, ETM)

Time: 3 hours Max. Marks: 80

Answer any five questions

All questions carry equal marks

- - -

1.a) Find v(t) for $t \geq 0$ and initial energy stored across a capacitor for the circuit shown in Fig.1. When the switch is opened at t = 0.

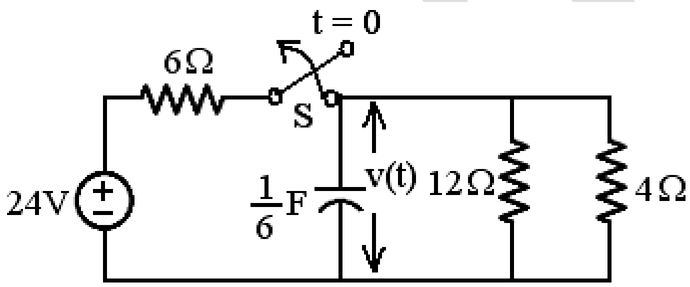


Fig.1

b) Determine the Laplace Transforms Impedance of the circuit shown in Fig.2. [8+7]

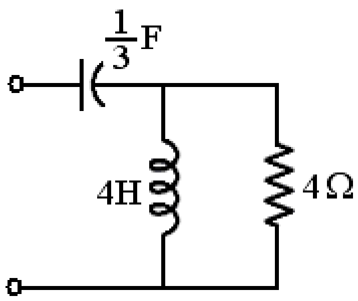


Fig.2

2. Find the Transmission parameters and Z – Parameters for the two – port network shown in Fig.3.

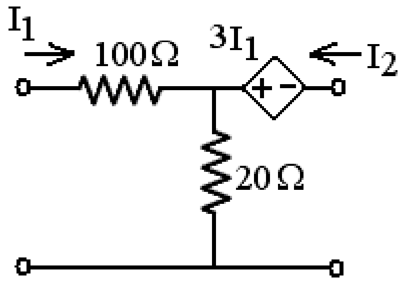


Fig.3

3. Design a band pass, constant – K filter with cut – off frequency of 4 KHz and nominal characteristic impedance of 500 Ω. [15]

4.a) Explain Symmetrical Bridge T – type attenuator.

b) Design a symmetrical bridge T – attenuator with attenuation of 20 dB and design impedance of 600 Ω.

5.a) Explain different types of dc generators with neat sketches and give the application of each.

b) A – 4 pole, lap wound armature when driven at 600 rpm generates 120V. If the flux per pole is 0.025 Wb, find the number of conductors on its armature. [7+8]

6.a) What are the various methods of speed control of dc shunt motor?

b) A 250 V, 10 kW shunt motor takes 2.5A when running light. The armature and field resistances are 0.3 Ω and 400 Ω respectively. Brush contact drop of 2V. Find the full – load efficiency of motor? [7+8]

7.a) Derive the expression for the induced emf of a Transformer.

b) A 6600/400V, 50 Hz, single phase Transformer has a net cross-sectional area of the core of 428 cm^2. The maximum flux density in the core is 1.5 Tesla. Calculate the number of turns in the primary and secondary windings. [7+8]

8. Explain the working principle of capacitor – start and capacitor – run single phase induction motors with the circuit diagram and also give their applications. [15]

B.Tech II Year - II Semester Examinations, December-2011 / January-2012

PRINCIPLES OF ELECTRICAL ENGINEERING

(COMMON TO ECE, ETM)

Time: 3 hours Max. Marks: 80

Answer any five questions

All questions carry equal marks

- - -

1.a) Find i(t) for t > 0 for the circuit shown in Fig.1. When the switch is opened at t = 0.

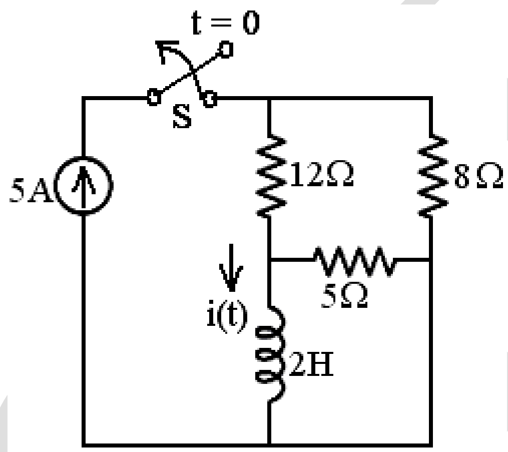


Fig.1

b) Determine the current i(t) for t ≥ 0, for the circuit shown in Fig.2. Assume initial conditions are zero. [10+5]

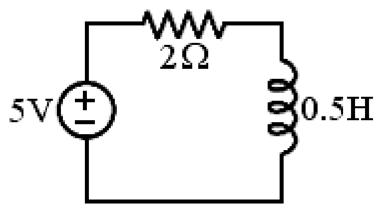


Fig.2

2. Determine the Z – Parameters and transmission parameters of the current shown in Fig.3. [15]

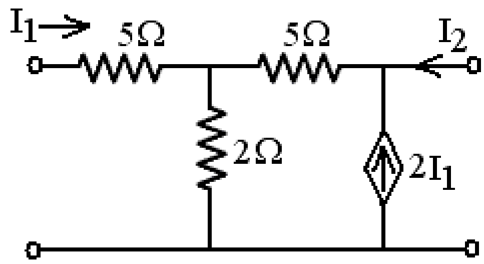


Fig.3

3. Design a low pass constant – K (i) T – Section and (ii) π – section filter with cut – off frequency (f_c) 6 kHz and nominal characteristic impedance of 500 Ω. [15]

4.a) Explain symmetrical lattice Attenuator.

b) Design a symmetrical lattice attenuator to have characteristic impedance of 600 Ω and attenuation of 20 dB. [7+8]

5. Explain in detail the Load characteristics of various DC generators with appropriate sketches and also give the applications of various generators. [15]

6.a) Derive the torque equation of a dc motor.

b) A 4 – pole, 220 V dc series motor has a wave connected armature with 1200 conductors. The flux per role is 20×10^{-3} wb, when the motor is drawing 46A. Armature and series field resistances are 0.25 Ω and 0.15 Ω respectively. Find

i) the speed ii) Total torque. [7+8]

7.a) Explain the importance of open circuit and short – circuit tests on a transformer.

b) Determine the full – load efficiency of 80 KVA, 1100/250V, 50Hz, Single phase transformer with iron losses of 800W and full – load copper losses of 400 W on LV side at unity power factor. [7+8]

8. Write short notes on the following:

a) Shaded – Pole motor

b) Capacitor motor

c) AC Servo motor. [15]

B.Tech II Year - II Semester Examinations, December-2011 / January-2012

PRINCIPLES OF ELECTRICAL ENGINEERING

(COMMON TO ECE, ETM)

Time: 3 hours Max. Marks: 80

Answer any five questions

All questions carry equal marks

- - -

1.a) Determine i, when the switch is opened at t = 0 for the circuit shown in Fig.1. Assume that the switch is closed for a long time.

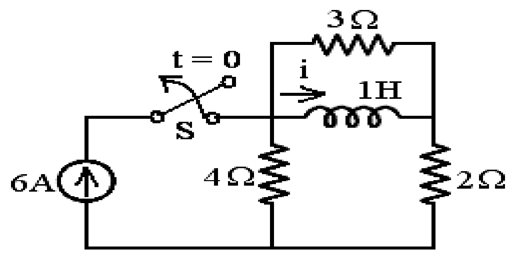


Fig.1

b) Determine the current i for t ≥ 0, if the initial current is zero, for the circuit shown in Fig.2. [7+8]

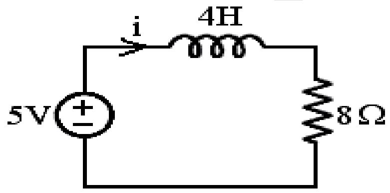
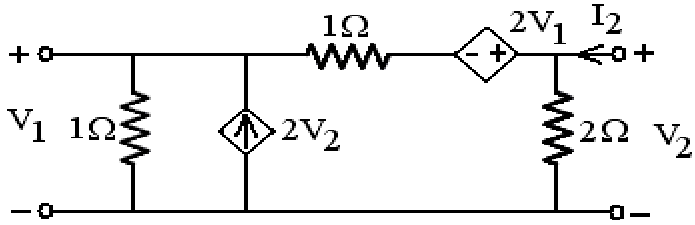


Fig.2

2. Determine h – parameters for the network shown in Fig.3. [15]



3.A high pass constant – K filter with cut – off frequency 40 kHz is required to procedure a maximum attenuation at 36 kHz when used with terminated resistance of 500 Ω. Design a suitable m – derived T – section. [15]

4.a) Draw the circuit of symmetrical lattice attenuation. Derive the design equation.

b) Design a symmetrical lattice attenuator to have attenuation of 20 dB and characteristic impedance of 500 Ω. [7+8]

5.a) Describe with suitable diagrams the principle of operation of a dc generator.

b) A 4 pole dc generator runs at 1000 rpm. Its armature is lap wound and has 740 conductors on its periphery. The useful flux per pole is 0.04 wb. Calculate the emf generated on open circuit. [8+7]

6.a) Explain the various losses which occurs in a dc motor.

b) A 500V dc shunt motor draws 4A on no load. The field current of the motor is 1.0A. Its armature resistance including brushes is 0.2 Ω. Find the efficiency, when the input current is 20A. [7+8]

7. Open circuit and short circuit tests conducted on a 10KVA, 500/2000V, 50 Hz, Single phase transformer gave the following readings:

OC Test: 500V, 120W on primary side.

SC Test: 15V, 20A, 100W on primary side.

Determine the efficiency on full load unity power factor. [15]

8. Write short notes on the following:

a) Capacitor – start motors.

b) Shaded pole motors

c) AC Tachometers

17.Question Bank

UNIT I:TRANSIENT ANALYSIS

Short Answer Questions:

1. What is meant by steady state condition?

2. Explain in brief about transient response of a network.

3. Explain the initial conditions of basic passive elements.

4. Give summary of passive elements under steady state ($t \to \infty$) and at ($t = 0^+$) Immediately after switching.

5. Obtain the expression for current i(t) for $t \geq 0$ in a driven series RL circuit with DC excitation, Hence obtain expression for $V_L(t)$, $V_R(t)$, $P_R(t)$ and $P_L(t)$.

6. Derive the expression for current i(t) for $t \geq 0$ in a undriven series RL circuit, Assume DC excitation, Also obtain $V_L(t)$.

7. What is meant by driven circuit and undriven circuit?

8. Derive expression for $V_C(t)$ for $t \geq 0$, for driven series RC circuit for DC excitation.
9. Obtain transient response of undriven or source free series RC circuit.

10. What is time constant? Explain the significance of time constant in case of RL series circuit and RC series circuit with DC excitation.

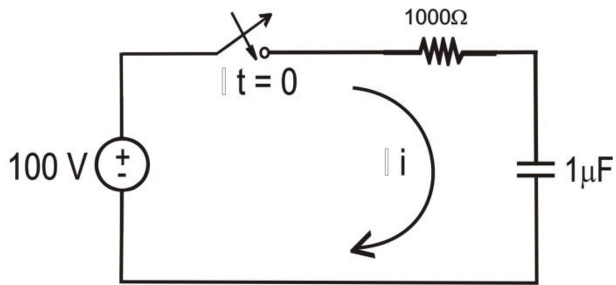
Long Answer Questions:

1. Derive the expression for transient response of a driven series RLC circuit for DC excitation. State different types of roots possible and write appropriate solution for the roots.

2. Define the following terms.
   a. Critical Resistance (R_C)
   b. Damping Ratio
   c. Natural Frequency (ω_n)
   d. Damped Frequency (ω_d)

3. A DC voltage of 20 V is applied in a series RL circuit, where R = 5 Ω and L = 10 H, Find

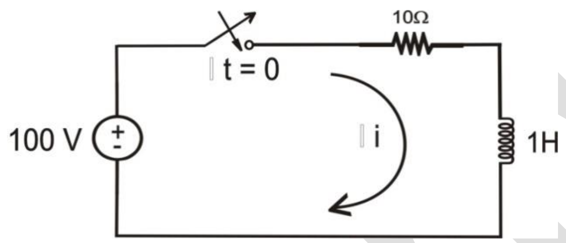
   a) Time Constant   b) Max Value of Stored Energy.



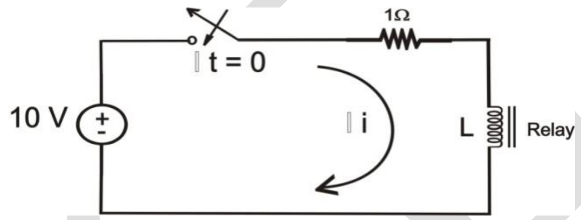
4. Switch is closed at t = 0, with the capacitor uncharged. Find the values of i, $\frac{di}{dt}, \frac{d^2i}{dt}$ at

t = 0^+.

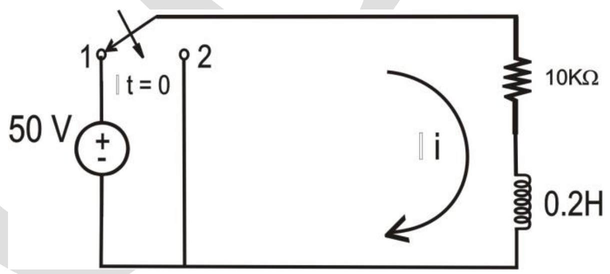
5.. Switch is closed at t = o, Assume initial current of inductor to be zero. Find the values of i, $\frac{di}{dt}, \frac{d^2i}{dt}$ at t = 0^+.



6. In the circuit shown, the relay is adjusted to operate at a current of 5 A at t = 0, switch is closed, The relay is found to operate at t = 0.347 sec. Find of inductance.



7.a.In a series RL circuit shown in fig. the switch is in position 1 for long time to establish a steady state and then moved to position2 at t = 0. Find the i(t) for t = 0.



b. A series RL circuit, with R = 30 Ω, L = 15 H, V = 60 (dc), applied at t = 0, determine I, V_R, V_L at transient state.

UNIT – II TWO PORT NETWORK PARAMETERS

Express the elements of a T-network in terms of the ABCD parameters.

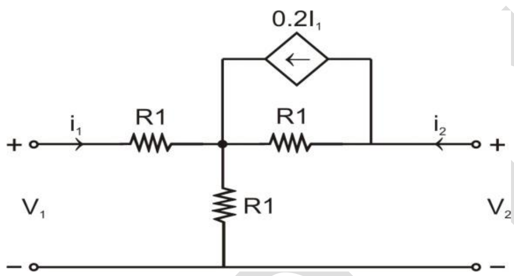
LONG ANSWER QUESTIONS

1. Obtain the expression for y-parameters in terms of transmission parameters.

(Nov./Dec-2004, Set – 1, May/June-2004, Set – 4)

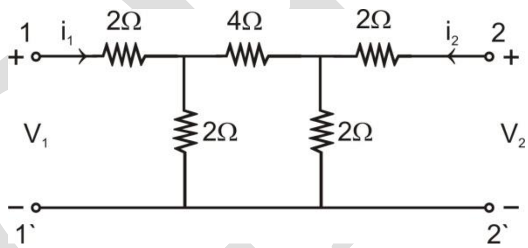
2. Find the π- equivalent circuit for the following two port network.

(May-2005, Set – 1, 8 Marks)



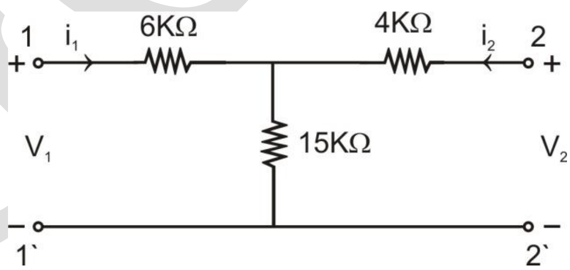
3. Obtain z-parameters for the network shown in the figure.

(Dec. 2005, Set – 4, June-2005, Set – 1, June 2004, Set – 2, 8 Marks)



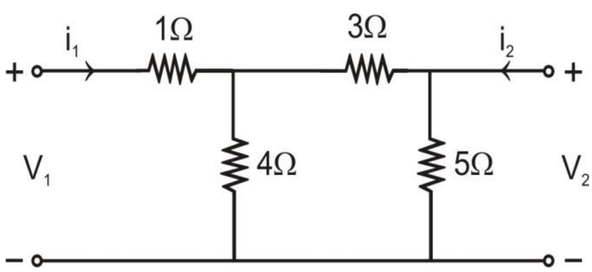
4. Find z-parameters of the network shown in the figure.

(Aug.-2006, Set – 4, June-2006, Set – 2, 8 Marks)



5. Determine the z-parameters of the network shown in the figure.

(June-2006, Set-1, 8 Marks)

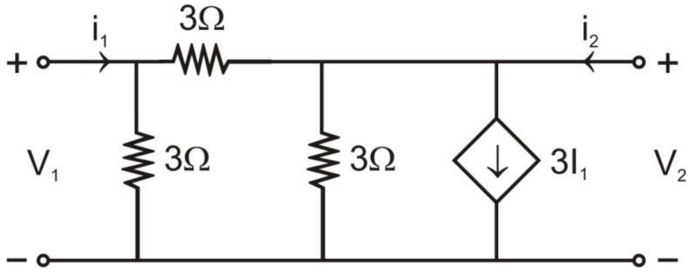


6. Determine y-parameters of the network shown in the figure.

(Aug.-2006, Set – 3, June-2006, Set – 4, 8

Marks)

7. The y-parameters of a two port network are as follows: Y_{11} = 0.6s, Y_{12} = -0.3s, Y_{22} = 1.2s.



Determine (i) ABCD parameters,

(ii) Equivalent π network.

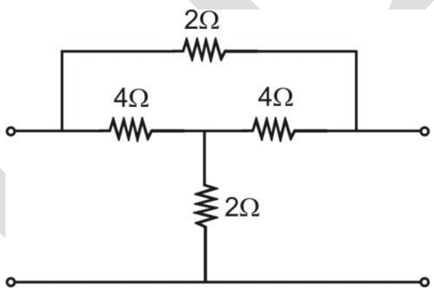
(June-2006, Set – 1, 8 Marks)

8. z - parameters for a two port network are given as follows Z_{11} = 25Ω, Z_{12} = Z_{21} = 20Ω, Z_{22} = 50Ω. Find the equivalent T network.

(June-2006, Set – 3, 8 Marks)

9. Obtain y-parameters of the following bridged T network.

(June-2004, Set – 4, Dec.-2004, Set – 1, 8 Marks)



UNIT III:FILTERS & ATTENUATORS

Long Answer Questions:

1. Design a band elimination filter having a design impedance of 600Ω and cut – off frequencies f_1 = 2 KHz and f_2 = 6 KHZ.

2. Explain T – type attenuator and also design a T – type attenuator to give an attenuation of 60dB and to work in a line of 500Ω impedance.

3. Design a m – derived high pass filter with a cut – off frequency of 10KHz; design impedance of 5Ω and m = 0.4.

4. Explain the lattice attenuator and also design a lattice attenuator to have a characteristic impedance of 800Ω and attenuation of 20 dB.

5. What is a constant – K low pass filter, derive its characteristics impedance.

6. Explain π – type attenuator and also design it to give 20db attenuation and to have characteristic impedance of 100Ω.

8. Design a band stop, constant – K filter with cut off frequencies of 4 KHz and 10 KHz and nominal characteristic impedance of 500 Ω.

9.a) Explain about a symmetrical π – attenuator.

b) Design a symmetrical π – attenuator to provide attenuation of 20dB and design impedance of and design impedance of 400 Ω

10. Design a band pass, constant – K filter with cut – off frequency of 4 KHz and nominal characteristic impedance of 500 Ω.

11.a) Explain Symmetrical Bridge T – type attenuator.

b) Design a symmetrical bridge T – attenuator with attenuation of 20 dB and design impedance of 600 Ω

UNIT IV:DC. MACHINES

Short Answer Questions:

1. What is a Machine?
2. What is a Generator?
3. Classify different types of Generators.
4. Classify different types of DC Generators.
5. What is a Motor?
6. Classify different types of Motors.
7. Classify different types of DC Motors.
8. State Fleming's right hand rule?
9. State Fleming's Left hand rule?
10. State various parts in a DC machine

Long Answer Questions:

1. Draw a detailed sketch of a d.c. machine and identify the different parts. Briefly explain the function of each major part?

2. a) Explain constructional features and working principle of d.c. generator?
   b) State Fleming's right hand rule?

3. a) Derive the expression of induced e.m.f. of d.c. generator?
   b) A 4 pole, lap wound d.c. generator has a useful flux of 0.07wb per pole. Calculate the generated e.m.f when it is rotated at a speed of 900r.p.m with the help of prime mover.

armature consists of 440 number of conductors. Also calculate the generated e.m.f if lap wound armature is replaced by wave wound armature.

4. Explain the characteristics of DC generators
5. Differentiate between slip rings and commutator in a d.c. machine?
6. What are the main parts of a d.c. machine? State the function of each part with relevant figures.
7. a) Based on the type of excitation classify the d.c. generators?
   b) A dynamo has a rated armature current of 250A.what is the current per path of the armature if the armature winding is lap or wave connected? The machine has 12 poles.
8. What is the construction and working principle of D.C motor?
9. Explain the different types of D.C motors and their characteristics?
10. Derive an expression for the speed of a D.C. motor in terms of back emf and flux per pole.
11. Explain speed current and speed torque characteristics of D.C. shunt motor.
12. What are the applications of a D.C motor and D.C generator?
13. Derive the expressions for various torques developed in a dc motor?
14. Explain the different methods of speed control of a dc shunt motor
15. What are the different losses occurring in a d c machine? Derive the condition for maximum efficiency of a dc motor
16. Explain the following
   a. Swinburne's test   b. Brake test

UNIT V:TRANSFORMERS & THEIR APPLICATIONS

Short Answer Questions:

1.Define tansformer.

2.Why is the rating of transformer in KVA and why not in KW?

3.Classify various types of transformers depending on their construction

4. Classify various types of transformers depending on their operation.

5. What is the construction and working principle of Transformer?

Long Answer Questions:

1. What is the construction and working principle of Transformer?
2. Derive an e.m.f. equation of a single phase transformer. The maximum flux density in the core of 250/3000 volts, 50 Hz single phase transformer is 1.2 webers per square meter. If the emf per turn is 8 volts determine primary and secondary turns and area of the core.

3. The primary winding of a 50 Hz single phase transformer has 480 turns and fed from 6400 v supply. The secondary winding has 20 turns. Find the peak value of flux in the core and the secondary voltage.

4. Derive an expression for voltage per turn of a transformer.

5. What are the different losses occurring in a transformer on load? How can these losses be determined experimentally?

6. Define the voltage regulation of a transformer. Deduce the expressions for the voltage regulation and the conditions for maximum voltage and zero voltage regulations.

7. The number of turns on the primary and secondary windings of a single phase transformer are 350 and 35 respectively. If the primary is connected to a 2.2kV,50 Hz supply, determine the secondary voltage.

8. Draw the phasor diagrams of a single phase transformer for the following load power factors

   a. Leading b. Leading c. Unity

9. Draw the equivalent circuits of a single phase transformer referred to primary as well as secondary

10. Explain double field revolving theory

12. Explain why a single phase motor is not self starting?

13. Explain the following with phasor diagrams

    a. Capacitor start and run motor b. Shaded pole motor

    c. Synchro's d. Stepper motor

    e. A C tachometers f. Servomotors

18.Assignment topics

UNIT I:TRANSIENT ANALYSIS

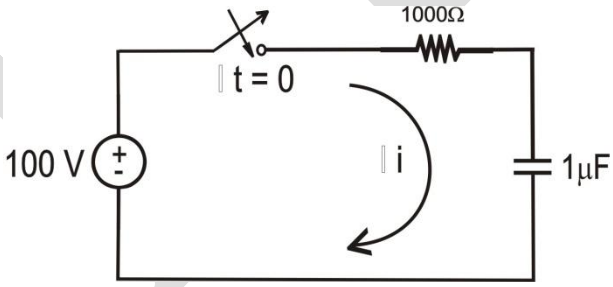
1. Derive the expression for transient response of a driven series RLC circuit for DC excitation. State different types of roots possible and write appropriate solution for the roots.

2. Define the following terms.
   e. Critical Resistance (R_C)
   f. Damping Ratio
   g. Natural Frequency (ω_n)
   h. Damped Frequency (ω_d)

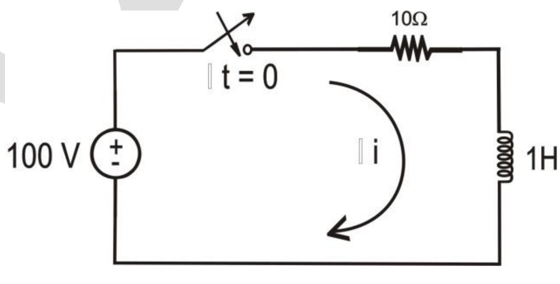
3. A DC voltage of 20 V is applied in a series RL circuit, where R = 5 Ω and L = 10 H, Find

   a) Time Constant b) Max Value of Stored Energy.

4. Switch is closed at t = 0, with the capacitor uncharged. Find the values of i, $\frac{di}{dt}, \frac{d^2i}{dt}$ at t = 0^+.



5.. Switch is closed at t = o, Assume initial current of inductor to be zero. Find the values of i, $\frac{di}{dt}, \frac{d^2i}{dt}$ at t = 0^+.



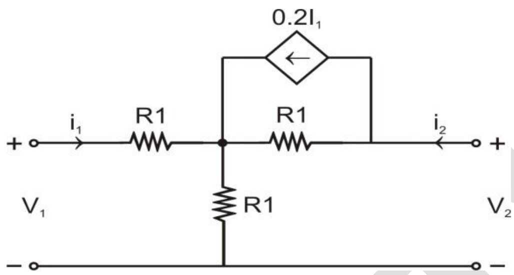
UNIT – II TWO PORT NETWORK PARAMETERS

1. Obtain the expression for y-parameters in terms of transmission parameters.

   (Nov./Dec-2004, Set – 1, May/June-2004, Set – 4)

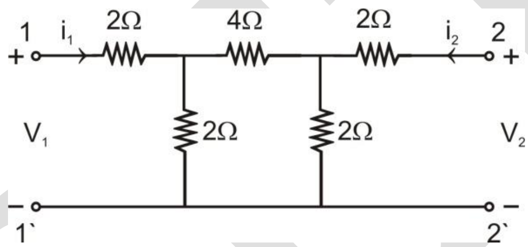
2. Find the π- equivalent circuit for the following two port network.

   (May-2005, Set – 1, 8 Marks)



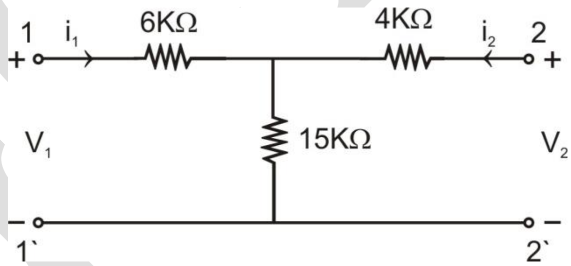
3. Obtain z-parameters for the network shown in the figure.

   (Dec. 2005, Set – 4, June-2005, Set – 1, June 2004, Set – 2, 8 Marks)



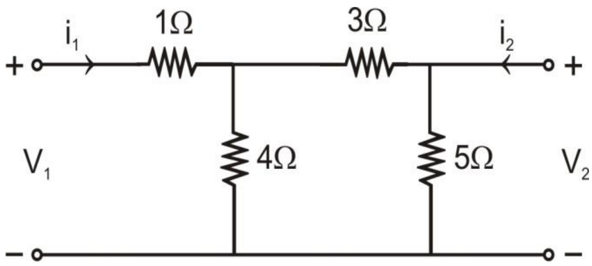
4. Find z-parameters of the network shown in the figure.

   (Aug.-2006, Set – 4, June-2006, Set – 2, 8 Marks)



5. Determine the z-parameters of the network shown in the figure.

   (June-2006, Set-1, 8 Marks)



(ii) Equivalent π network.

UNIT III:FILTERS & ATTENUATORS

1. Design a band elimination filter having a design impedance of 600Ω and cut – off frequencies f_1 = 2 KHz and f_2 = 6 KHZ. [15]

2. Explain T – type attenuator and also design a T – type attenuator to give an attenuation of 60dB and to work in a line of 500Ω impedance. [15]

3. Design a m – derived high pass filter with a cut – off frequency of 10KHz; design impedance of 5Ω and m = 0.4. [15]

4. Explain the lattice attenuator and also design a lattice attenuator to have a characteristic impedance of 800Ω and attenuation of 20 dB. [15]

UNIT IV:DC. MACHINES

1. Draw a detailed sketch of a d.c. machine and identify the different parts. Briefly explain the function of each major part?

2. a) Explain constructional features and working principle of d.c. generator?

   b) State Fleming's right hand rule?

3. a) Derive the expression of induced e.m.f. of d.c. generator?

   b) A 4 pole, lap wound d.c. generator has a useful flux of 0.07wb per pole. Calculate the generated e.m.f when it is rotated at a speed of 900r.p.m with the help of prime mover. armature consists of 440 number of conductors. Also calculate the generated e.m.f if lap wound armature is replaced by wave wound armature.

4. What is the construction and working principle of D.C motor?

5. Explain the different types of D.C motors and their characteristics?

6. Derive an expression for the speed of a D.C. motor in terms of back emf and flux per pole.

UNIT V:TRANSFORMERS & THEIR APPLICATIONS

1. What is the construction and working principle of Transformer?

2. Derive an e.m.f. equation of a single phase transformer. The maximum flux density in the core of 250/3000 volts, 50 Hz single phase transformer is 1.2 webers per square meter. If the emf per turn is 8 volts determine primary and secondary turns and area of the core.

3. The primary winding of a 50 Hz single phase transformer has 480 turns and fed from 6400 v supply. The secondary winding has 20 turns. Find the peak value of flux in the core and the secondary voltage.

4. Derive an expression for voltage per turn of a transformer.

5. What are the different losses occurring in a transformer on load? How can these losses be determined experimentally?

19. Unit Wise Objective Questions:

UNIT I:TRANSIENT ANALYSIS

1. Laplace transform analysis gives
   a. time domain response only
   b. Frequency domain response only.
   c. Both a and b options.

2. Match the following :

(i) Undamped	a) $\xi = 0$
(ii) Under damped	b) $\xi = 1$
(iii) Critically damped	c) $1 < \xi < \infty$
(iv) Over damped	d) $0 < \xi < 1$
	e) $1 > \xi > \infty$
	f) $\xi = \sqrt{-1}$

3. Match the following:

(i) Critical Resistance (R_C)	a) $\frac{R}{2}\sqrt{C/L}$
(ii) Damping ratio (ξ)	b) $\omega_n\sqrt{1-\xi^2}$
(iii) Natural frequency (ω_n)	c) $\frac{1}{LC}$
(iv) Damping frequency (ω_d)	d) $2\sqrt{L/C}$
	e) $1/RC$
	f) $1/\sqrt{LC}$

4. The time constant of below network is ________________________ seconds.

5. Match the following

(i) Time const of series RL Circuit is	a) only memory (L, C) elements

(ii) Time const of series RC circuit is	b) only memoryless ® elements
(iii) Transient response occur's in	c) RC
(iv) Inductor do not allow sudden	d) $\frac{1}{RC}$
(v) Capacitor do not allow sudden	e) changes in currents.
(vi) Capacitor do not allow sudden	f) changes in voltages
	g) $\frac{L}{R}$
	h) $\frac{R}{L}$

UNIT – II TWO PORT NETWORK PARAMETERS

1. An attenuator is a

(A) R's network. (B) RL network.

(C) RC network. (D) LC network.

Ans: A

2. For a two port reciprocal network, the three transmission parameters are given by A = 4, B = 7 and C = 5. The value of D is equal to

(A) 8.5 (B) 9

(C) 9.5 (D) 8

Ans: B

3.A symmetrical T network has characteristic impedance Z and propagation constant oγ . Then the series element Z1 and shunt element Z2 are given by

(A) Z = Z sinh γ and Z 2Z tanh γ

(B) Z = Z sinh γ and Z 2Z tanh 2γ

(C) Z 2Z tan 2 γ and Z = Z sinh γ

(D) Z Z tanh 2 γ and Z = 2Z sinh γ

Ans: C

4 For a linear passive bilateral network

(A) h21 = h12   (B) h21 = −h12   (C) h12 = g12   (D) h12 = −g12

Ans: B

5. For a symmetrical network

(A) Z11 = Z22 (B) Z12 = Z21

(C) Z11 = Z22 and Z12 = Z21 (D) Z11 x Z22 – $Z12^2$=0

Ans: C

6. Bridged T network can be used as:

(A) Attenuator (B) Low pass filter

(C) High pass filter (D) Band pass filter

Ans: A

UNIT III:FILTERS & ATTENUATORS

1.The Characteristic Impedance of a low pass filter in attenuation Band is

(A) Purely imaginary. (B) Zero.

(C) Complex quantity. (D) Real value.

Ans: A

2. The purpose of an Attenuator is to:

(A) increase signal strength. (B) provide impedance matching.

(C) decrease reflections. (D) decrease value of signal strength.

Ans: D

3. In a transmission line terminated by characteristic impedance, Zo

(A) There is no reflection of the incident wave.

(B) The reflection is maximum due to termination.

(C) There are a large number of maximum and minimum on the line.

(D) The incident current is zero for any applied signal.

Ans: A

4. All pass filter

(A) passes whole of the audio band.

(B) passes whole of the radio band.

(C) passes all frequencies with very low attenuation.

(D) passes all frequencies without attenuation but phase is changed.

Ans: D

5. If ' 'α is attenuation in nepers then

(A) attenuation in dB = α / 0.8686. (B) attenuation in dB = 8.686 α.

(C) attenuation in dB = 0.1 α. (D) attenuation in dB = 0.01 α.

Ans: B

6. For an m-derived high pass filter, the cut off frequency is 4KHz and the filter has an

infinite attenuation at 3.6 KHz, the value of m is

(A) 0.436 (B) 4.36

(C) 0.34 (D) 0.6

Ans: A

7.In a variable bridged T-attenuator, with , RA = Ro zero dB attenuation can be obtained if bridge arm RB and shunt arm R are set as C

(A) R B = ,0 RC = ∞ (B) 0 RB = ∞,RC =

(C) R B = ,R RC = ∞ (D) RB = ,0 RC = R

Ans: A

8.A constant K band-pass filter has pass-band from 1000 to 4000 Hz. The resonance frequency of shunt and series arm is a

(A) 2500 Hz. (B) 500 Hz.

(C) 2000 Hz. (D) 3000 Hz.

Ans: C

9.A constant k low pass T-section filter has Z0 = 600Ω at zero frequency. At f = fc the characteristic impedance is

(A) 600Ω (B) 0

(C) ∞ (D) More than 600Ω

Ans: B

10. In m-derived terminating half sections, m =

(A) 0.1 (B) 0.3

(C) 0.6 (D) 0.95

Ans: C

11. In the m-derived HPF, the resonant frequency is to be chosen so that it is

(A) above the cut-off frequency. (B) Below the cut-off frequency.

(C) equal to the cut-off frequency. (D) None of these.

12. An A transmission line works as an

(A) Attenuator (B) LPF

(C) HPF (D) Neither of the above

Ans: B

UNIT IV:DC MACHINES

1. A machine that converts mechanical energy into electrical energy of d.c in nature is called
   a) a.c motor b) d.c. generator
   c) a.c generator d) d.c motor
2. The basic principle of working of a d.c. generator is by
   a) Faradays law of electromagnetic induction b) Maxwells cork screw rule
   c) Flemings left hand rule d) Amperes thumb rule
3. The basic essential parts of electrical generator are
   a) Magnetic field and insulator b) Electric field and insulator
   c) Magnetic field and conductor d) Electric field and conductor
4. The direction of induced e.m.f in a generator depends upon the direction of

a) Electric field b) Magnetic field

c) motion of the conductor d) none of the above

5. The direction of the motion of conductor in generator is given by

a) Flemings left hand rule b) Flemings right hand rule

c) Amperes thumb rule d) Maxwells cork screw rule

6. The armature of the d.c .generator is made up of cast iron or cast steel because

a) it has to provide low reluctance path b) it has to provide high reluctance path

c) both d) none

7. Which of the following is applicable to a d.c. machine with respect to the field windings?

a) It is always placed on stator b) It is always placed on rotor

c) Sometimes on rotor d) May be on rotor or stator.

8. The armature core of d.c. machine is made up of __________

a) solid aluminum b) laminated aluminum

c) solid steel d) laminated steel.

9. Which of the following is function of the brushes in case of d.c. machine?

a) To convert a.c. to d.c. b) To convert to d.c. to a.c.

c) To collect current and deliver to the load

d) May be conversion for a.c. to d.c. or d.c. to a.c.

10. The number of commutator segments must be __________

a) twice the number of armature coils b) equal to the number of armature coils

c) half the number of armature coils d) thrice the number of armature coils.

11. The direction of generated e.m.f. in d.c. generator is determined by _________

a) Lenz's law b) Faraday's law

c) Fleming's left hand rule d) Fleming's right hand rule.

12. Which of the following gives the expression for the generated voltage in a d.c. generator?

a) $4.44\ \phi$ Z NP b) $4.44/\phi$ Z NP

c) ϕ Z NP / A 60 d) zero.

13. Which of the following forms of energy conversion take place in an electrical energy system?

a) Mechanical to electrical b) Electrical to mechanical

c) Mechanical to thermal d) Thermal to Mechanical

14. Which of the following is a function of the commutator in d.c. generator?

a) to act as a rectifier b) to act as a inverter

c) to act as a junction box per connection the armature winding ends.

d) to act as a chopper.

15. A shunt generator cannot excite, if the field resistance is ________ critical value.

a) less than
b) more than
c) equal to
d) none

16. Laminations are used in d.c. machine to reduce

a) eddy current losses
b) Hysteresis losses
c) copper losses
d) none

17. In a cumulative compound generator flux produced by shunt field winding and series field winding ---------------- each other

a) aids
b) opposes
c) nullifies
d) none

18. In a differential compound generator flux produced by shunt field winding and series field winding ------------ each other

a) aids
b) opposes
c) nullifies
d) none

19. The field winding is also called as

a) exciting winding
b) armature winding
c) both
d) none

20. Brushes are normally made up of soft material like

a) carbon
b) aluminum
c) cast steel
d) cast iron

21. A d.c motor is used to……….

a) generate power
b) change mechanical energy to electrical energy
c) change electrical energy to mechanical energy
d) increase energy put into it

22. A d.c motor is still used in industrial applications because it…..

a) is cheap
b) is simple in construction
c) provides fine speed control
d) none of the above

23. Carbon brushes are preferable to Copper brushes because….

a) they have longer life
b) they reduce armature reaction
c) they have lower resistance
d) they reduce sparking

24. The field poles and armature of d.c machine are laminated to…..

a) reduce the weight of the machine
b) decreases the speed
c) reduce eddy currents
d) reduce armature reaction

25. The back e.m.f in a d.c motor………

a) oppose the applied voltage
b) aids the applied voltage
c) adds the armature current
d) none of the above

26. The value of back e.m.f (E_b) in a d.c motor is maximum at…..

a) no load b) full load c) half full load d) none of the above

27. The motor is equation is given by……

a) $V = E_b - I_a R_a$ b) $V = E_b + I_a R_a$ c) $E_b = I_a R_a - V$ d) None of the above

28. The mechanical power developed in a d.c motor is maximum when back e.m.f(E_b) is equal to……….the applied voltage (V)

a) twice b) half c) one-third d) none of the above

29. When the speed of a d.c motor increases its armature current….

a) increases b) decreases c) remains constant d) none of the above

30. The amount of emf of a shunt motor will increase when…..

a) the load increase b) the field is weakend

c) the field is strengthened d) none of the above

31. The speed of d.c motor is ………..

a) directly proportional to flux per pole b) inversely proportional to flux per pole

c) inversely proportional to applied voltage d) none of the above

32. The torque developed by a d.c motor is directly proportional to…..

a) flux per pole * armature current b) armature resistance * applied voltage

c) armature resistance * armature current d) none of the above

33. Armature reaction in d.c motor is increased…………

a) when the armature current increases b) when the armature current decreases

c) when the field current increases d) by interpoles

34. W.r.t the direction of rotation interpoles on a d.c motor must have the same polarity as the main poles …………

a) ahead of them b) behind of them c) in between them d) none of them

35. In a d.c motor the brushes shifted from the mechanical neutral plain in a direction opposite to the rotation……

a) decrease speed b) increase speed c) reduce sparking d) produce flat characteristics

36. In very large d.c motors with severe heavy duty armature reaction effects are corrected by

a) using interpoles only b) using compensatory windings in addition to interpoles

c) shifting the brush position d) none of the above

37. The speed of a …… motor is practically constant

a) cumulatively compounded b) series c) differentially compounded d) shunt

38. In DC shunt motors as load is reduced

a) The speed will increase abruptly

b) The speed will increase in proportion to reduction in load

c) The speed will remain almost constant d) The speed will reduce

39. What will happen if the back emf of a DC motor vanishes

a) The motor will stop   b) The motor will continue to run

c) The armature may burn   d) The motor will run noisy

40. Small DC motors up to 5HP usually have

a) 2 poles   b) 4 poles   c) 6 poles   d) 8 poles

UNIT V:TRANSFORMERS & THEIR APPLICATIONS

1. A transformer will work on _________

a) a.c only   b) d.c only

c) a.c as well as d.c   d) none of the above

2. The primary and secondary of a transformer are ………………. Coupled

a) electrically   b)magnetically

c) electrically & magnetically   d) none of the above

3. A transformer is an efficient device because it ……………..

a) is a static device   b) uses inductive coupling

c) Uses capacitve coupling   d) Uses electric coupling

4. The voltage per turn of the primary of transformer is……. The voltage for turn of the secondary

a) more than   b) less than

c) the same as   d) none of the above

5. The iron core is used to…….. of the transformer

a) increase the weight   b) provide tight magnetic coupling

c) reduce core losses   d) none of the above

6. The maximum flux produced in the core of a transformer

a) directly proportional to supply frequency

b) inversely proportional to supply frequency

c) inversely proportional to primary voltage

d) none of the above

7. When the primary of a transformer is connected to a d.c supply…………

a) primary draws small current

b) primary leakage reactance is increased

c) core losses are increased

d) primary may burn out

8. An ideal transformer is one which………

a) has no losses and leakage reactance

b) does not work

c) as same number of primary and secondary turns

d) none of the above

9. A transformer has an efficiency of 80% and works at 100V, 4KW if the secondary voltage 240V,find the primary current

(a) 40A (b) 30A (c) 20A (d) 10A

10. In the above question, what is the secondary current

a) 12.5A b) 9.42A c) 11.56A d) 13.33A

11. A 2000/200V, 20KVA ideal transformer has 66 turns in the secondary the no. of primary turns is……………

a) 440 b) 660 c) 550 d) 330

12. The no-load ratio of a 50Hz single phase transformer is 6000/250V the maximum flux in the core is 0.06Wb. What is the no of primary turns.

a) 450 b) 900 c) 350 d) 210

13. In the above question what is the no. of secondary turns?

a) 38 b) 19 c) 76 d) 104

14. A 20 turn iron cored indicator is connected to a 100V, 58Hz source. The maximum flux density in the core is 1Wb/m^2. The cross sectional area of the core is……….

a) 0.152m^2 b) 0.345 m^2 c) 0.056 m^2 d) 0.0225 m^2

15. Calculate the core area required for a 1600kVA, 6600/440V, 50Hz single phase core type power transformer. Assume a maximum flux density of 1.2 WB/m^2 and induced voltage per turn of 30 V.

a) 975 cm^2 b) 1100 cm^2 c) 1125 cm^2 d) 1224 cm^2

16. An ideal transformer

a) Has no losses and magnetic leakage

b) Has interleaved primary and secondary windings

c) Has common core for its primary and secondary windings

d) Has core of stain less steel and windings of pure copper metal

17. The phase relationship between primary and secondary terminal voltage of a Transformer is

a) Primary voltage is leading the secondary voltage by 90^o

b) Secondary voltage is leading the primary voltage by 90^o

c) 180^o out of phase d) In the same phase

18. If an ammeter in the secondary of a 100/10 V transformer reads 10 A. What would be the current in the primary

a) 1 A b) 2 A c) 10 A d) 100 A

19. The %age voltage regulation of the Transformer is given by

a) $\frac{E_2 - V_2}{V_2}$ b) $\frac{E_2 - V_2}{E_2}$ c) $\frac{V_2 - E_2}{E_2}$ x 100 d) $\frac{E_2 - V_2}{E_2}$ x 100

20. The full load rating of a Transformer is 90 kW at power factor of 0.9 its KVA rating would be

21. The stator of a 3-phase induction motor produces________ magnetic field
    a) steady   b)rotating
    c) alternating   d) None of the above

22. An induction motor is preferred to a d.c motor because it __________
    a) Provides high starting torque   b) Provides smooth speed control
    c) has simple and rugged construction   d)none of the above

23. A 3-phase induction motor is_____________
    a) Essentially a constant speed   b) a variable speed motor
    c) Very costly   d) Not easily maintainable

24. If the frequency of 3- phase supply to the stator of a 3-phase induction motor is increased, then synchronous speed___________
    a) is decreased   b) is increased
    c) Remains un changed   d) None of the above

25. The synchronous speed of a 3-phase induction motor having 20-poles and frequency 50Hz is
    a) 600rpm   b)100rpm
    c) 1200rpm   d)300rpm

26. The relation among synchronous speed (N_S) rotor speed (N) and slip(S) is____________
    a) $N=N_S (S-1)$ b) $N=N_S (1-S)$
    c) $N=N_S (S+1)$ d) $N=N_S S$

27. When the rotor of a 3- phase induction motor is blocked, the slip___________
    a) 0   b) 0.5   c) .1   d) 1

28. A 4-pole induction motor has a synchronous speed of 1500 r.p.m then supply frequency
    a) 50Hz   b)25Hz
    c) 60Hz   d)none of the above

29. The rotor winding of a 3-phase wound rotor induction motor is generally________ connected
    a) Star   b) delta
    c) partly star and partly delta   d) none of the above

30. A wound rotor motor is mainly used in applications where__________
    a) High starting torque   b) speed control is required
    c) less costly motor is not required   d)high rotor resistances required

31. If the slip of a 3-phase induction motor increases, the p.f .of the circuit
    a) is increased   b) is decreased   c) remains unchanged d) none of the above

32. Which of the following is drawback of the Induction Motor

a) cheap in cost
b) moderate efficiency
c) self starting
d) speed control is complex

33. The frequency of induced e.m.f in case of rotor

a) sf
b) f/s
c) f+s
d)f-s

34. The copper losses in the rotor of induction motor

a) result in the eddy currents
b) are lost as heat
c) result in noise
d) are always negligible

35. The ratio of resistance to reactance for induction motor is

a) high
b) unity
c) less than unity
d) negligible

36. Power factor of induction motor during no load condition is

a) low
b) high
c) zero
d) unity

37. Which of the following is a rotational transformer

a) transformer
b) D.C machine
c) Induction motor
d) synchronous machine

38. An induction motor is

a) non self starting
b) self starting with low torque
c) self starting with high torque
d) self starting with zero torque

39. At low slip the torque slip characteristic is

a) $T \propto S$
b) $T \propto S^2$
c) $T \propto \frac{1}{S^2}$
d) $T \propto \frac{1}{S}$

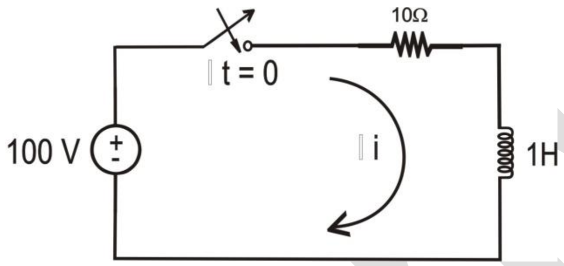
40. The relationship between rotor frequency f_2, slip s and stator frequency f_1 is given by

a) $f_2 = Sf$
b) $f_2 = \sqrt{Sf}$
c) $f_2 = f / S$
d) $f_2 = (1-S)f$

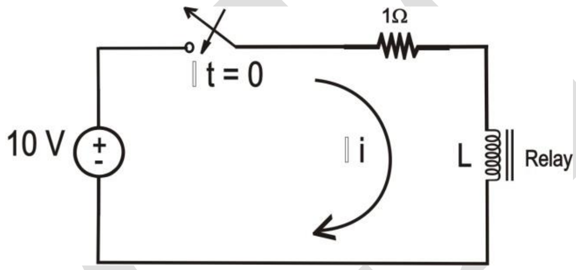
20. Tutorial problems

UNIT I:TRANSIENT ANALYSIS

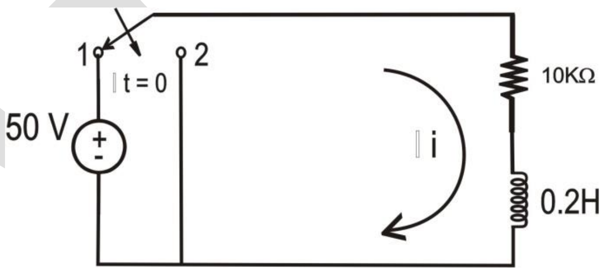
1.. Switch is closed at t = o, Assume initial current of inductor to be zero. Find the values of i, $\frac{di}{dt}, \frac{d^2 i}{dt}$ at t = 0^+.



2. In the circuit shown, the relay is adjusted to operate at a current of 5 A at t = 0, switch is closed, The relay is found to operate at t = 0.347 sec. Find of inductance.



3.a.In a series RL circuit shown in fig. the switch is in position 1 for long time to establish a steady state and then moved to position2 at t = 0. Find the i(t) for t = 0.

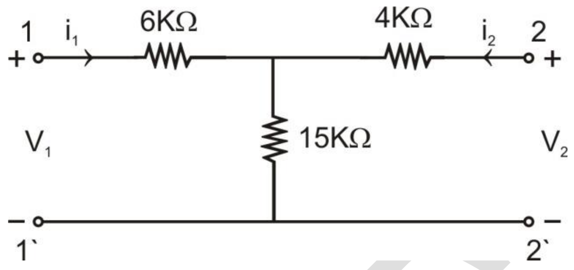


c. A series RL circuit, with R = 30 Ω, L = 15 H, V = 60 (dc), applied at t = 0, determine I, V_R, V_L at transient state.

UNIT – II TWO PORT NETWORK PARAMETERS

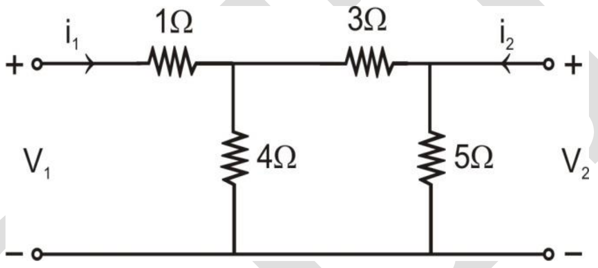
1. Express the elements of a T-network in terms of the ABCD parameters.
2. Find z-parameters of the network shown in the figure.

(Aug.-2006, Set – 4, June-2006, Set – 2, 8 Marks)



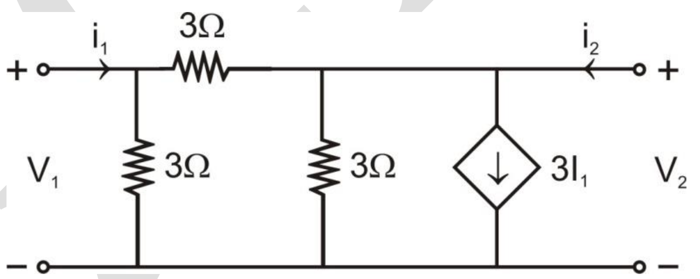
3. Determine the z-parameters of the network shown in the figure.

(June-2006, Set-1, 8 Marks)



4. Determine y-parameters of the network shown in the figure.

(Aug.-2006, Set – 3, June-2006, Set – 4, 8 Marks)



5. The y-parameters of a two port network are as follows: Y_{11} = 0.6s, Y_{12} = -0.3s, Y_{22} = 1.2s.

   Determine (i) ABCD parameters,

   (ii) Equivalent π network.

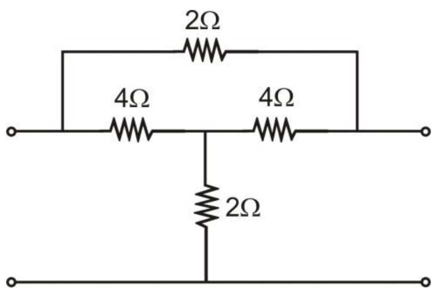
(June-2006, Set – 1, 8 Marks)

6. z - parameters for a two port network are given as follows Z_{11} = 25Ω, Z_{12} = Z_{21} = 20Ω, Z_{22} = 50Ω. Find the equivalent T network.

(June-2006, Set – 3, 8 Marks)

7. Obtain y-parameters of the following bridged T network.

(June-2004, Set – 4, Dec.-2004, Set – 1, 8 Marks)



UNIT III:FILTERS & ATTENUATORS

1. What is a constant – K low pass filter, derive its characteristics impedance.

2. Explain π – type attenuator and also design it to give 20db attenuation and to have characteristic impedance of 100Ω.

3. Design a band stop, constant – K filter with cut off frequencies of 4 KHz and 10 KHz and nominal characteristic impedance of 500 Ω.

4.a) Explain about a symmetrical π – attenuator.

b) Design a symmetrical π – attenuator to provide attenuation of 20dB and design impedance of and design impedance of 400 Ω

5. Design a band pass, constant – K filter with cut – off frequency of 4 KHz and nominal characteristic impedance of 500 Ω.

6.a) Explain Symmetrical Bridge T – type attenuator.

b) Design a symmetrical bridge T – attenuator with attenuation of 20 dB and design impedance of 600 Ω

UNIT IV:DC. MACHINES

1. Explain the characteristics of DC generators
2. Differentiate between slip rings and commutator in a d.c. machine?
3. What are the main parts of a d.c. machine? State the function of each part with relevant figures.
4. a) Based on the type of excitation classify the d.c. generators?
   b) A dynamo has a rated armature current of 250A.what is the current per path of the armature if the armature winding is lap or wave connected? The machine has 12 poles.
5. Derive the expressions for various torques developed in a dc motor?
6. Explain the different methods of speed control of a dc shunt motor
7. What are the different losses occurring in a d c machine? Derive the condition for maximum efficiency of a dc motor
8. Explain the following
   a. Swinburne's test b. Brake test

UNIT V:TRANSFORMERS & THEIR APPLICATIONS

1. Derive an e.m.f. equation of a single phase transformer. The maximum flux density in the core of 250/3000 volts, 50 Hz single phase transformer is 1.2 webers per square meter. If the emf per turn is 8 volts determine primary and secondary turns and area of the core.
2. The primary winding of a 50 Hz single phase transformer has 480 turns and fed from 6400 v supply. The secondary winding has 20 turns. Find the peak value of flux in the core and the secondary voltage.
3 The number of turns on the primary and secondary windings of a single phase transformer are 350 and 35 respectively. If the primary is connected to a 2.2kV,50 Hz supply, determine the secondary voltage.
4. Draw the phasor diagrams of a single phase transformer for the following load power factors

   a. Leading b. Leading c. Unity

5. Draw the equivalent circuits of a single phase transformer referred to primary as well as secondary
6. a. Capacitor start and run motor b. Shaded pole motor

   c. Synchro's d. Stepper motor

   e. A C tachometers f. Servomotors

21. Known gaps if any

NIL

22. Discussion questions

UNIT1:

1) Derive expression for transient response of a driven series RLC circuit for DC excitation.State different types of roots possible and write appropriate solution for the roots
2)What is time constant?Explain the significance of time constant in case of series RC circuit
3)Obtain transient response of undriven or sourse free series RC CIRCUIT

UNIT 2:

1) Obtain the expression for y-parameters in terms of transmission parameters.
2) Define h parameters and draw the equivalent circuit for the same
3) Derive condition of symmetry for ABCD parameters

UNIT 3:

1) Explain about a symmetrical π – attenuator
2) Design a band stop, constant – K filter with cut off frequencies of 4 KHz and 10 KHz and nominal characteristic impedance of 500 Ω. [15]
3) Explain Symmetrical Bridge T – type attenuator.

UNIT 4:

1)Derive the expressions for various torques developed in a dc motor?

2) Explain constructional features and working principle of d.c. generator?

UNIT5:

1) Draw the equivalent circuits of a single phase transformer referred to primary as well secondary
2) Derive the expression for the induced emf of a transformer

23. References, Journals, websites and E-links

1Electric circuits- A.Chakrabarthy, Dhanipat Rai & Sons.

2.Basic concepts of Electrical Engineering- PS Subramanyam, BS Publications

3.Engineering Circuit Analysis – W.H.Hayt and J. E. Kermmerly and S. M. Durbin 6 ed., 2008 TMH.

4.Basic Electrical Engineering- S.N.Singh, PHI.

6. Electrical Circuits- David A.Bell, Oxford University Press.
7. Electric Circuit Analysis- K.S.Suresh Kumar, Pearson Education.
8. Electrical Circuits- N.Sreenivasulu.

24:Quality sheets:

To be attached

25.Student List

To be attached

26. GroupWise Student List for discussion topics

To be attached